

## SECTION – A (1 MARK EACH)

1. If  $a$  and  $b$  are two positive integers such that  $a = 14b$ . Find the HCF of  $a$  and  $b$ .  
**Ans:** We can write  
 $a = 14b + 0$   
 $\therefore$  remainder is 0  
 $\therefore$  HCF is  $b$ .
2. If zeroes of  $p(x) = 2x^2 - 7x + k$  are reciprocal of each other, then find the value of  $k$ .  
**Ans:**  
 $\therefore$  Zeroes are reciprocal of each other  
 $\therefore$  Product of zeroes = 1  $\Rightarrow \frac{k}{2} = 1 \Rightarrow k = 2$
3. Find the value of  $k$  so that the following system of equations has no solution:  $3x - y - 5 = 0$ ,  $6x - 2y + k = 0$ .  
**Ans:** Here  $a_1 = 3$ ,  $b_1 = -1$ ,  $c_1 = -5$ ,  
and  $a_2 = 6$ ,  $b_2 = -2$ ,  $c_2 = k$ .  
For no solution,  
 $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \Rightarrow \frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{k} \Rightarrow \frac{1}{2} \neq \frac{-5}{k} \Rightarrow k \neq -10$
4. For which values of  $p$ , does the pair of equations given below has unique solution?  $4x + py + 8 = 0$  and  $2x + 2y + 2 = 0$ .  
**Ans:** For unique solution,  
 $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{4}{2} \neq \frac{p}{2}$   
 $\Rightarrow p \neq 4$   
Therefore, for all values of  $p$  other than 4, the given pair of equations has unique solution.
5. The HCF of two numbers is 145 and their LCM is 2175. If one number is 725, then find the other number.  
**Ans:** HCF  $\times$  LCM = Product of two numbers  
 $\Rightarrow 145 \times 2175 = 725 \times \text{other number}$   
 $\therefore$  Other number =  

$$\frac{145 \times 2175}{725}$$
 $= 435$
6. For what value of  $k$ , are the roots of the quadratic equation  $3x^2 + 2kx + 27 = 0$  real and equal.  
**Ans:**  $D = b^2 - 4ac \Rightarrow D = (2k)^2 - 4 \times 3 \times 27 = 4k^2 - 324$   
For real and equal roots,  $D = 0 \Rightarrow 4k^2 - 324 = 0 \Rightarrow 4k^2 = 324 \Rightarrow k^2 = 81 \Rightarrow k = \pm 9$ .

## SECTION – B (2 MARKS EACH)

7. 4 Bells toll together at 9.00 am. They toll after 7, 8, 11 and 12 seconds respectively. How many times will they toll together again in the next 3 hours?

**Ans:**  $7 = 7 \times 1,$

$8 = 2 \times 2 \times 2,$

$11 = 11 \times 1,$

$12 = 2 \times 2 \times 3,$

$\therefore \text{LCM of } 7, 8, 11, 12 = 2 \times 2 \times 2 \times 3 \times 7 \times 11 = 1848$

$\therefore$  Bells will toll together after every 1848 sec.

$\therefore$  In next 3 hrs, number of times the bells will toll together =

$$\frac{3 \times 3600}{1848}$$

= 5.84

$\Rightarrow$  5 times.

8. Solve for  $x$  and  $y$ :  $2x + 3y = 7$ ;  $4x + 3y = 11$

**Ans:**

Given equation are

$2x + 3y = 7 \dots(i)$

$4x + 3y = 11 \dots(ii)$

Here coefficients of  $y$  in both the equations are equal

$\therefore$  Subtracting (ii) from (i) we get

$$\begin{array}{r} 2x + 3y = 7 \\ 4x + 3y = 11 \\ \hline -2x = -4 \end{array}$$

$x = 2$

when  $x = 2$ , equation (i) becomes

$2 \times 2 + 3y = 7 \Rightarrow 3y = 3 \Rightarrow y = 1$

$\therefore x = 2, y = 1$

### SECTION – C (3 MARKS EACH)

9. Prove that  $\sqrt{3}$  is irrational.

Let  $\sqrt{3}$  be a rational number

Therefore,  $\sqrt{3} = p/q$  where  $p, q$  are co-primes and  $q \neq 0$

On squaring both sides, we get  $p^2 = 3q^2 \dots(1)$

$\Rightarrow 3$  is a factor of  $p^2$  [since,  $3q^2 = p^2$ ]  $\Rightarrow 3$  is a factor of  $p$

Let  $p = 3m$  for all  $m$  ( where  $m$  is a positive integer)

Squaring both sides, we get  $p^2 = 9m^2 \dots(2)$

From (1) and (2), we get  $3q^2 = 9m^2 \Rightarrow q^2 = 3m^2$

$\Rightarrow 3$  is a factor of  $q^2$  [since,  $q^2 = 3m^2$ ]  $\Rightarrow 3$  is a factor of  $q$

Thus, we see that both  $p$  and  $q$  have common factor 3 which is a contradiction that  $p, q$  are co-primes.

Therefore, Our assumption is wrong

Hence  $\sqrt{3}$  is not a rational number i.e., irrational number.

10. Solve for  $x$ :  $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}; a \neq 0, b \neq 0, x \neq 0$

**Ans:**

$$\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x} \Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$$

$$\frac{x-a-b-x}{x(a+b+x)} = \frac{b+a}{ab} \Rightarrow \frac{-(a+b)}{x(a+b+x)} = \frac{a+b}{ab}$$

$\Rightarrow x(a+b+x) = -ab \Rightarrow x^2 + (a+b)x + ab = 0$

$\Rightarrow (x+a)(x+b) = 0 \Rightarrow x = -a, x = -b$

11. On a morning walk three persons step off together and their steps measure 40 cm, 42 cm, 45cm, what is the minimum distance each should walk so that each can cover the same distance incomplete steps?

**Ans:**

Minimum distance = LCM of 40, 42 and 45

$$\text{Now } 40 = 2^3 \times 5$$

$$42 = 2 \times 3 \times 7$$

$$45 = 3^2 \times 5$$

$$\therefore \text{LCM of 40, 42 and 45} = 2^3 \times 3^2 \times 5 \times 7 = 2520$$

$\therefore$  They should walk 2520 cm or 25.20 m to cover the distance in complete steps.

12. If  $\alpha$  and  $\beta$ , are zeroes of polynomial  $p(x) = 5x^2 + 5x + 1$  then find the value of (i)  $\alpha^2 + \beta^2$   
(ii)  $\alpha^{-1}$  and  $\beta^{-1}$

Given polynomial is

$$p(x) = 5x^2 + 5x + 1$$

$$\text{Here } a = 5, b = 5, c = 1$$

$$(i) \text{ Now } \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$\Rightarrow \alpha^2 + \beta^2 = \left(\frac{-b}{a}\right)^2 - 2\frac{c}{a}$$

$$[\because \text{sum of zeroes i.e. } \alpha + \beta = \frac{-b}{a} \text{ and product of zeroes i.e. } \alpha\beta = \frac{c}{a}]$$

$$\Rightarrow \alpha^2 + \beta^2 = \left[\frac{-5}{5}\right]^2 - 2 \times \frac{1}{5} = 1 - \frac{2}{5} = \frac{3}{5}$$

$$(ii) \alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$$

$$= \frac{-b/a}{c/a} = \frac{-5/5}{1/5} = -5$$

### SECTION – D (4 MARKS EACH)

13. Solve the following system of linear equations graphically:  $3x + y - 12 = 0$ ;  $x - 3y + 6 = 0$   
Shade the region bounded by the lines and  $x$ -axis. Also, find the area of shaded region.

**Ans:**

Table for  $3x + y - 12 = 0$

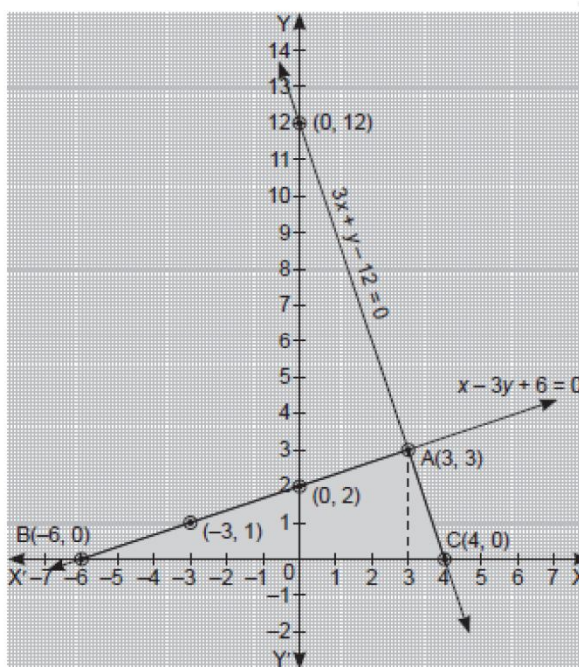
x	0	4	2
y	12	0	6

Table for  $x - 3y + 6 = 0$

x	-6	0	-3
y	0	2	1

$\Delta ABC$  is the region bounded by the lines and  $x$ -axis.

$$\text{Area } \Delta ABC = \frac{1}{2} BC \times AD = \frac{1}{2} \times 10 \times 3 = 15 \text{ sq. units}$$



14. A motor boat whose speed is 18 km/h in still water takes 1 hr. more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream.

Ans:

Speed of boat = 18 km/h

Let speed of stream be  $x$  km/h

Upstream speed be  $(18 - x)$  km/h

Downstream speed be  $(18 + x)$  km/h

Time taken to cover 24 km upstream =  $\frac{24}{(18 - x)}$  hours

Time taken to cover 24 km downstream =  $\frac{24}{(18 + x)}$  hours

$$\text{ATQ } \frac{24}{(18 - x)} - \frac{24}{(18 + x)} = 1$$

$$\Rightarrow 24(18 + x) - 24(18 - x) = (18)^2 - (x)^2$$

$$\Rightarrow 432 + 24x - 432 + 24x = 324 - x^2$$

$$\Rightarrow x^2 + 48x - 324 = 0$$

$$\Rightarrow x^2 + 54x - 6x - 324 = 0$$

$$\Rightarrow x(x + 54) - 6(x + 54) = 0$$

$$\Rightarrow (x - 6)(x + 54) = 0$$

$$\Rightarrow x = 6, -54 \text{ (rejecting) } (\because \text{Speed cannot be in -ve})$$

$$\therefore \text{Speed of stream} = 6 \text{ km/h}$$

**CASE STUDY-BASED QUESTIONS. EACH QUESTION CARRIES 1 MARK**

15. A test consists of 'True' or 'False' questions. One mark is awarded for every correct answer while  $\frac{1}{4}$  mark is deducted for every wrong answer. A student knew answers to some of the questions. Rest of the questions he attempted by guessing. He answered 120 questions and got 90 marks.

- (i) If answer to all questions he attempted by guessing were wrong, then how many questions did he answer correctly?
- (ii) How many questions did he guess?
- (iii) If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got?
- (iv) If answer to all questions he attempted by guessing were wrong then how many questions answered correctly to score 95 marks?

Ans:

Let number of questions whose answer is known to the student be  $x$ .

and questions attempted by cheating be  $y$

A.T.Q.

$$\therefore x + y = 120 \dots(i)$$

Marks got = 90

$$\Rightarrow x - \frac{1}{4}y = 90 \Rightarrow 4x - y = 360 \dots(ii)$$

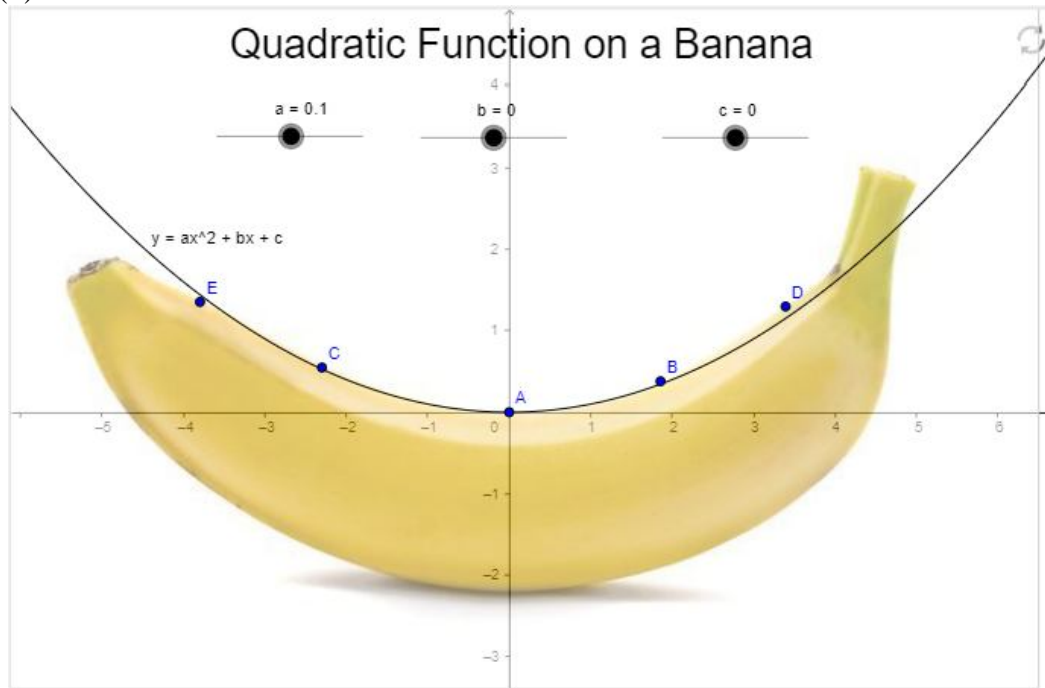
Solving (i) and (ii), we get

$$x = 96 \text{ and } y = 24$$

$\therefore$  He answer correctly = 96 questions.

- (i) He answered 96 questions correctly.
- (ii) He attempted 24 questions by guessing.
- (iii) Marks =  $80 - (1/4)x \ 40 = 80 - 10 = 70$
- (iv) Let the number of questions answered correctly be  $x$  then number of answered by guessing is  $120 - x$  then we have
$$x - (1/4)x(120 - x) = 95$$
$$\Rightarrow 5x/4 = 95 + 30 = 125$$
$$\Rightarrow 5x = 500$$
$$\Rightarrow x = 100$$

16. The below quadratic function can model the natural shape of a banana. Now, we know that a parabolic shape must have a quadratic function, therefore an equation in standard form of  $f(x)=ax^2 + bx + c$ . To find an equation for the parabolic shape of the banana, we need to find the values of a, b, and c. From the banana picture above, we can see that a quadratic function is able to model the banana quite accurately, with  $a=0.1$ ,  $b=0$ , and  $c=0$ . Therefore, the equation is  $f(x)=0.1x^2$ .



- (i) Name the shape of the banana curve from the above figure.  
 Ans: Parabola
- (ii) Find the number of the zeroes of the polynomial for the shape of the banana.  
 Ans: No. of zeroes = 1
- (iii) If the curve of banana represented by  $f(x) = x^2 - x - 12$ . Find its zeroes.  
 Ans:  $x^2 - x - 12 = x^2 - 4x + 3x - 12 = 0$   
 $\Rightarrow x(x - 4) + 3(x - 4) = 0$   
 $\Rightarrow (x - 4)(x + 3) = 0$   
 $\Rightarrow x = 4, -3$  are the required zeroes.
- (iv) If the representation of banana curves whose one zero is 4 and the sum of the zeroes is 0 then find the quadratic polynomial.  
 If one zero is 4 and the sum of zeroes is 0 then the other zero is  $-4$ .  
 Product =  $4 \times -4 = -16$   
 Required quadratic polynomial is  $x^2 - 16$