KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS HYD – 32

SAMPLE TEST PAPER 04 FOR CLASS X (2020-21)

SAMPLE ANSWER

Max. marks: 80 **Time Allowed: 3 hrs**

General Instruction:

1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part – A:

1. It consists three sections- I and II.
2. Section I has 16 questions of 1 mark each. Internal choice is provided in 5 questions.
3. Section II has 4 questions on case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B:

1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

PART - A

SECTION-I

Questions 1 to 16 carry 1 mark each.

1. Check whether 307 is a term of the list of numbers 6, 13, 20, 27, …..

Here, $13 - 6 = 20 - 13 = 27 - 20 = 7$.

So, the given progression is an AP with $a = 6$ and $d = 7$.

Let 307 be the nth term of the A.P.

Then $a_n = 307 \Rightarrow a + (n - 1)d = 307$

$\Rightarrow 6 + (n - 1)(7) = 307 \Rightarrow 7n - 7 = 307 - 6 \Rightarrow 7n = 301 + 7$

$\Rightarrow 7n = 308 \Rightarrow n = \frac{308}{7} = 44$

$\therefore$ 307 is 44th term of the given A.P.

OR

Find how many two-digit numbers are divisible by 9.

The list of two digits numbers divisible by 9 is 18, 27, 36, 45, ... 99.

Here $a = 18$, $d = 27–18 = 9$, $a_n = 99$

Now $a_n = 99 \Rightarrow a + (n - 1)d = 99$

$\Rightarrow 18 + (n - 1)(9) = 99$

$\Rightarrow (n - 1)(9) = 99 - 18 \Rightarrow 9(n - 1) = 81$

$\Rightarrow (n - 1) = \frac{81}{9} = 9 \Rightarrow n = 9 + 1 \Rightarrow n = 10$

Hence, there are 10 two digit numbers divisible by 9.

2. Find the zeroes of $p(x) = x^2 – 27$.

To find a zeroes of $p(x)$, we equate $p(x)$ to zero.

i.e. $p(x) = 0 \Rightarrow x^2 - 27 = 0$

$\Rightarrow x^2 = 27 \Rightarrow x = \pm\sqrt{27} \Rightarrow x = \pm 3\sqrt{3}$

3. The cost of 3 pens and 5 pencil boxes is Rs. 120 and that of 4 pens and 7 pencil boxes is Rs. 250. Represent the situation algebraically.

Let ₹x be the cost of a pen and ₹y be the cost a pencil box.

$\therefore\ 3x + 5y = 120$

$4x + 7y = 250$

4. Find the values of m for which system 3x + my = 1, 2x – 7y = 5 will have a unique solution.

Here $a_1 = 3$, $b_1 = m$, $c_1 = -1$ and $a_2 = 2$, $b_2 = -7$, $c_2 = -5$

For unique solution, we have, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \Rightarrow \frac{3}{2} \neq \frac{m}{-7}$

$\Rightarrow m \neq \frac{-21}{2}$

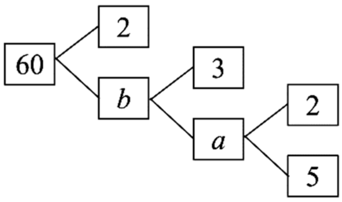
5. The decimal expansion of the rational number $\frac{23}{2^5 \times 5^4}$ will terminate after how many places of decimals?

The denominator $\frac{23}{2^5 \times 5^4}$ is $2^5 \times 5^4$, in which power of 2 is maximum i.e. 5.

$\therefore$ The given number will terminate after 5 places of decimal.

OR

Find the missing numbers in the following factor tree:



$a = 2 \times 5 = 10$

$b = 3 \times a = 3 \times 10 = 30$

6. For what value of m, the quadratic equation $x^2 - mx + 9 = 0$ have real roots?

Here, $a = 1$, $b = -m$, $c = 9$

Now, $D = b^2 - 4ac = (-m)^2 - 4 \times 1 \times 9$

$\Rightarrow m^2 - 36 = m^2 - 6^2 = (m + 6)(m - 6)$

For real roots, $D \geq 0$

$\Rightarrow (m + 6)(m - 6) \geq 0 \Rightarrow m \geq 6$ or $m \leq -6$

7. Find the discriminant of the quadratic equation $3x^2 - 5x + 7 = 0$ and hence find the nature of its roots.

Here $a = 3$, $b = -5$, $c = 7$

So, discriminant (D) $= b^2 - 4ac = (-5)^2 - 4(3)(7) = 25 - 84 = -59$

$\because$ D < 0, so, roots are not real.

OR

Find the roots of the equation $x^2 - 5x + 6 = 0$.

$x^2 - 5x + 6 = 0 \Rightarrow x^2 - 2x - 3x + 6 = 0$

$\Rightarrow x(x - 2) - 3(x - 2) = 0$

$\Rightarrow (x - 2)(x - 3) = 0$

$\Rightarrow x - 2 = 0$ or $x - 3 = 0$

$\Rightarrow x = 2$ or $x = 3$

Hence, the roots are 2 and 3.

8. If $4\sec\theta = 5$, then find the value of $\sin\theta$.

$4\sec\theta = 5 \Rightarrow \cos\theta = \frac{4}{5}$

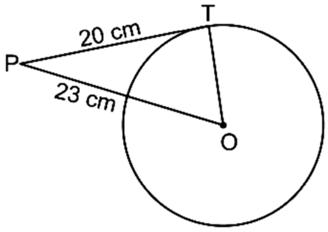
$\therefore \sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$.

9. A point P is 23 cm away from the centre of a circle and the length of tangent drawn from P to the circle is 20 cm. Find the radius of the circle.

Let O be the centre of the circle and P be a point outside the circle such that OP = 23 cm. Let PT be the tangent to the circle.

Then PT = 20 cm. Join OT.

Since the radius through the point of contact is perpendicular to the tangent, we have $\angle OTP = 90°$



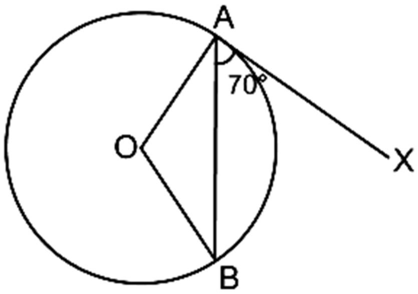
In ΔOTP, we have,

$OT^2 = OP^2 - PT^2 = (23)^2 - (20)^2$

$= (23 + 20)(23 - 20) = 43 \times 3 = 129$

$\Rightarrow OT = \sqrt{129}$ cm

10. If O is the centre of a circle, AB, is a chord and the tangent AX at A makes an angle of 70° with AB. Find AOB.

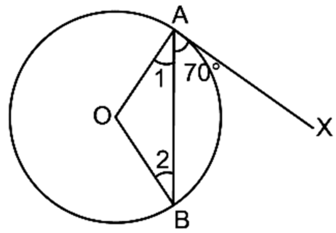


$\angle OAX = 90^0$ [AX is a tanget to the circle at A with centre O]

$\Rightarrow \angle 1 + \angle BAX = 90°$

$\Rightarrow \angle 1 + 70° = 90° \Rightarrow \angle 1 = 20°$

Also OA = OB [Radii of same circle]



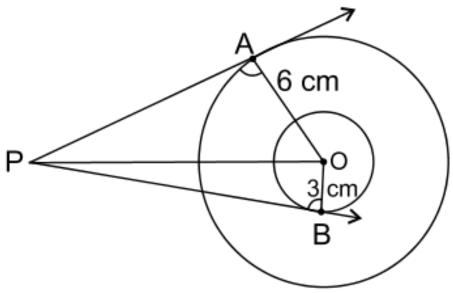
$\angle 1 = \angle 2 = 20°$ [Angle opposite to equal sides of a triangle are equal]

In ΔAOB, $\angle 1 + \angle 2 + \angle AOB = 180°$

$\Rightarrow \angle AOB = 180° - 20° - 20° = 140°$

OR

In the below figure, if AP = 10 cm, AO = 6cm and OB = 3 cm, then find BP.



AP = 10 cm [Given]

$\therefore$ In ΔOAP, $AP^2 + OA^2 = OP^2 \Rightarrow OP^2 = 100 + 36 \Rightarrow OP = \sqrt{136}$

In ΔOBP, $OP^2 = PB^2 + OB^2 \Rightarrow 136 = 9 + PB^2 \Rightarrow PB = \sqrt{127}$

11. Find the area of the sector of a circle with radius 7 cm and of central angle 60°.

Area of the sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{60°}{360°} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2 = \frac{1}{3} \times 11 \times 7 \text{ cm}^2 = 25.67 \text{ cm}^2.$

12. Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r.

Radius of cone = radius of the hemisphere = r.

Height of cone = radius of the hemisphere = r

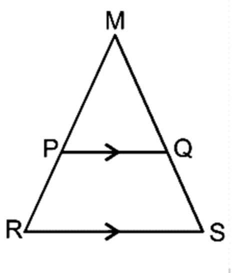
$\therefore$ Required volume = $\frac{1}{3}\pi r^2\ h = \frac{1}{3}\pi r^3$ cubic units.

13. In ΔMRS, PQ || RS and $\frac{MP}{PR} = \frac{2}{3}$. If MS = 4.5 cm, find MQ.

In ΔMRS, PQ || RS.

Let MQ = x.

$\therefore$ Using BPT, we have

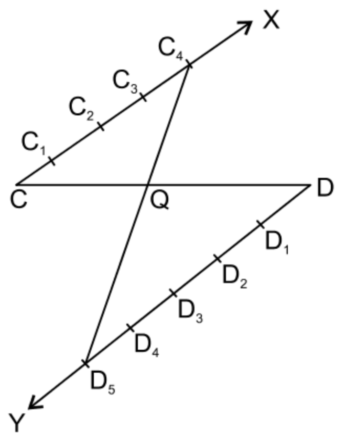


$$\frac{MP}{PR} = \frac{MQ}{QS} \Rightarrow \frac{2}{3} = \frac{x}{4.5 - x}$$

$\Rightarrow 9 - 2x = 3x \Rightarrow 9 = 5x$

$\Rightarrow x = \frac{9}{5}$ = 1.8 cm $\Rightarrow$ MQ = 1.8 cm

14. In the below figure, if C_1, C_2, C_3, ... and D_1, D_2, D_3, ... have been marked at equal distances. In what ratio Q divides CD?



Ans: 4 : 5

15. If p = 3 $\sec^2\theta$ and q = 3$\tan^2\theta$ – 1, then find p – q.

$p = 3\ \sec^2\theta$ and $y = 3\ \tan^2\theta - 1$

$p - q = 3\ \sec^2\theta - (3\ \tan^2\theta - 1)$

$= 3(1 + \tan^2\theta) - 3\ \tan^2\theta + 1$

$= 3 + 3\ \tan^2\theta - 3\ \tan^2\theta + 1$

$= 3 + 1 = 4$

16. Find the probability of getting a non-face card from a well shuffled deck of 52 playing cards.

Total no. of cards = 52

Total no. of face cards = 12

So, no. of non-face cards = 52 – 12 = 40

$\therefore$ P(a non-face card) = $\frac{40}{52} = \frac{10}{13}$.

OR

A boy calculates that the probability of his winning the first prize in a lottery is 7/100. How many tickets has he bought is 2000 tickets are sold?

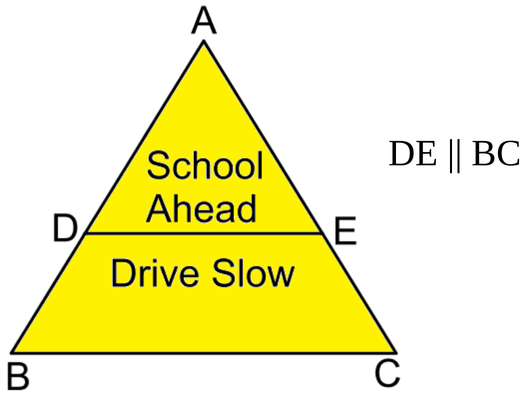
No of tickets bought = $\frac{7}{100} \times 2000 = 140$.

SECTION-II

Case study based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. Case Study based-1: Safety Board

A group of students to volunteer are working in making a safety board for school. They prepared once triangular safety board for their school with title "School Ahead" and "Drive Slow" in two parts of the triangular board as shown in below figure.



(a) If AD = 2 cm, BD = 5 cm and AE = 3 cm, then EC = ?

(i) $\frac{15}{2}$ (ii) $\frac{3}{5}$ (iii) $\frac{1}{5}$ (iv) $\frac{6}{5}$

Ans: (i) $\frac{15}{2}$

(b) If AD = 3 cm, Ab = 9 cm, BC = 6 cm, then DE = ?

(i) 4 cm (ii) 3 cm (iii) 1 cm (iv) 2 cm

Ans: (iv) 2 cm

(c) If ∠A = 60^o and ∠ADE = 50^o, then ∠C = ?

(i) 70^o (ii) 75^o (iii) 85^o (iv) 40^o

Ans: (i) 70^o

(d) Which of the following is correct?

(i) ΔADE ~ ΔABC (ii) ΔADE ≅ ΔABC (iii) Both (i) and (ii) (iv) none of these

Ans: (i) ΔADE ~ ΔABC

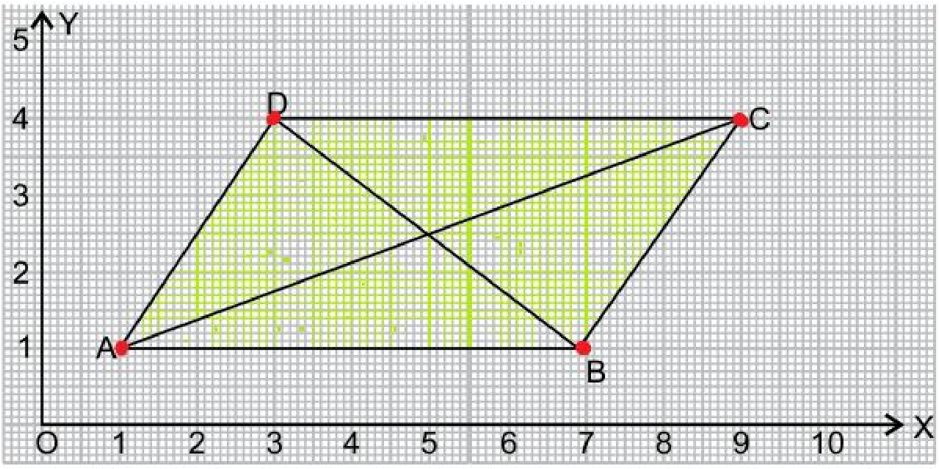
(e) What is the ratio of ar(ΔADE) to ar(ΔABC)?

(i) AD^2/AB^2 (ii) AD/AB (iii) 1 (iv) none of these

Ans: (i) AD^2/AB^2

18. Case Study based-2:

One day, Mohan visited his friend's apartment. From his balcony, he observed that there is flower bed on the ground which is in the shape of a parallelogram. Four red colour poles are there at the corners of the garden. He draws the sketch of the flower bed on a graph paper as shown in below figure.



(a) The coordinates of the vertex D are

(i) (3, 4) (ii) (4, 3) (iii) (3, 3) (iv) (4, 4)

Ans: (i) (3, 4)

(b) The coordinates of the point of intersection of the diagonals are:

(i) (5, 5) (ii) (5/2, 5/2) (iii) (5, 5/2) (iv) (5/2, 5)

Ans: (iii) (5, 5/2)

(c) The length of the side AB is:

(i) 7 units (ii) 6 units (iii) 5 units (iv) none of these

Ans: (ii) 6 units

(d) The length of the side AD is

(i) √13 units (ii) 13 units (iii) 14 units (iv) √14 units

Ans: (i) √13 units

(e) If we take A as the origin and AB as x-axis then the cooridnates of M are

(i) (4, 3/2) (ii) (3/2, 4) (iii) (4,4) (iv) (3/2, 3/2)

Ans: (i) (4, 3/2)

19. Case Study based-3:

A scale drawing of an object is the same shape at the object but a different size. The scale of a drawing Is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio. The ratio of two corresponding sides in similar figures is called the scale factor

$$\text{Scale factor} = \frac{\text{length in image}}{\text{corresponding length in object}}$$

If one shape can become another using revising, then the shapes are similar. Hence, two shapes are similar when one can become the other after a resize, flip, slide or turn.

In the photograph below showing the side of a train engine. Scale is 1 : 200.

This means that a length of 1 cm on the photograph above corresponds to a length of 200 cm, or 2 metres, on the actual engine. The scale can also be written as the ratio of two lengths.

(a) The overall length of the engine in the photograph above, including the couplings if the length of the model is 11 cm is :

(i) 22 cm (ii) 220 cm (iii) 220 m (iv) 22 m

Ans: (iv) 22 m

(b) What will affect the similarity of any two polygons?

(i) They are flipped horizontally

(ii) They are dilated by a scale factor

(iii) They are translated down

(iv) They are not the mirror image of one another

Ans: (iv) They are not the mirror image of one another

(c) What is the actual width of the door if the width of the door in photograph is 0.35 cm?

(i) 0.7 m (ii) 0.7 cm (iii) 0.07 cm (iv) 0.07 m

Ans: (i) 0.7 m

(d) If two similar triangles have a scale factor of 5 : 3, which statement regarding the two triangles is true?

(i) The ratio of their perimeters is 15 : 1

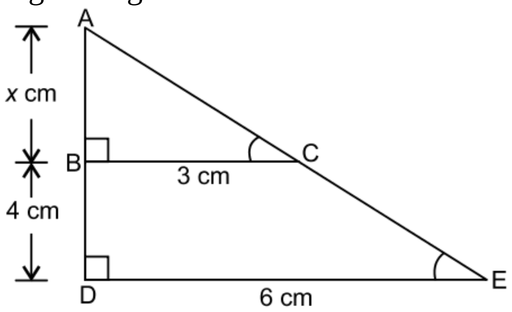
(ii) Their altitudes have a ratio 25 : 15

(iii) Their medians have a ratio 10 : 4

(iv) Their angle bisectors have a ratio 11 : 5

Ans: (ii) Their altitudes have a ratio 25 : 15

(e) The length of AB in the given figure is :

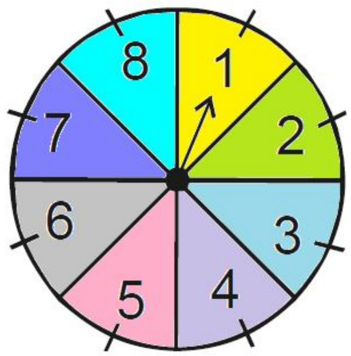


(i) 8 cm (ii) 6 cm (iii) 4 cm (iv) 10 cm

Ans: (iii) 4 cm

20. Case Study based-4:

One day Rahul visited park along with his friend. There he saw a game of chance that consists of spinning an arrow (as shown in below figure) that comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes.



(a) Find the probability that the arrow will point at 2.

(i) $\frac{1}{2}$ (ii) $\frac{1}{8}$ (iii) $\frac{3}{8}$ (iv) $\frac{5}{8}$

Ans: (ii) $\frac{1}{8}$

(b) Find the probability that the arrow will point at an even number.

(i) $\frac{1}{2}$ (ii) $\frac{1}{8}$ (iii) $\frac{3}{8}$ (iv) $\frac{1}{4}$

Ans: (i) $\frac{1}{2}$

(c) Find the probability that the arrow will point at a prime number.

(i) $\frac{1}{2}$ (ii) $\frac{1}{8}$ (iii) $\frac{3}{8}$ (iv) $\frac{5}{8}$

Ans: (i) $\frac{1}{2}$

(d) Find the probability that the arrow will point at a number divisible by 3.

(i) $\frac{1}{2}$ (ii) $\frac{1}{8}$ (iii) $\frac{3}{8}$ (iv) $\frac{1}{4}$

Ans: (iv) $\frac{1}{4}$

(e) Find the probability that the arrow will point at a number greater than 2.

(i) $\frac{1}{2}$ (ii) $\frac{1}{8}$ (iii) $\frac{3}{4}$ (iv) $\frac{1}{4}$

Ans: (iii) $\frac{3}{4}$

PART – B

(Question No 21 to 26 are Very short answer Type questions of 2 mark each)

21. Find the value of m, if the distance between the points X(–2, –12) and Y(m, –4) is 8 units.

$$XY = \sqrt{(m+2)^2 + (-4+12)^2} = \sqrt{m^2 + 4 + 4m + 64}$$

$$\Rightarrow 8 = \sqrt{m^2 + 4m + 68}$$

$$\Rightarrow m^2 + 4m + 68 = 64 \Rightarrow m^2 + 4m + 4 = 0$$

$$\Rightarrow m^2 + 2m + 2m + 4 = 0 \Rightarrow m(m + 2) + 2(m + 2) = 0$$

$$\Rightarrow (m + 2)(m + 2) = 0 \Rightarrow m + 2 = 0 \Rightarrow m = -2$$

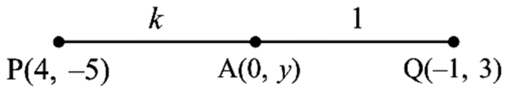
OR

Find the ratio in which the x-axis divides the line segment joining the points (4, –5) and (–1, 3).

Let A(O, y) divide the line segment joining the points P(4, –5) and Q(–1, 3) in the ratio $k : 1$.

Now coordinates of A are

$$\left(\frac{k(-1)+1(4)}{k+1}, \frac{k(3)+1(-5)}{k+1}\right)$$



i.e., $\left(\frac{-k+4}{k+1}, \frac{3k-5}{k+1}\right)$

Since point A lies on the x–axis, so ordinate = 0

$$\Rightarrow \frac{3k-5}{k+1} = 0 \Rightarrow 3k - 5 = 0 \Rightarrow k = \frac{5}{3}.$$

Hence, required ratio is $k : 1 = \frac{5}{3} : 1 = 5 : 3$.

22. Show that 12^n cannot end with digit 0 or 5 for any natural number n.

Expressing 12 as the product of primes, we obtain

$12 = 2^2 \times 3$

$\Rightarrow 12^n = (2^2 \times 3)^n = (2)^{2n} \times 3^n$

So, only primes in the factorisation are 2 and 3 and not 5

Hence, 12^n cannot end with digit 0 or 5.

23. If α and β are the zeroes of the polynomial $f(x) = x^2 - 5x + k$ such that If $\alpha - \beta = 1$, find the value of k.

$f(x) = x^2 - 5x + k$

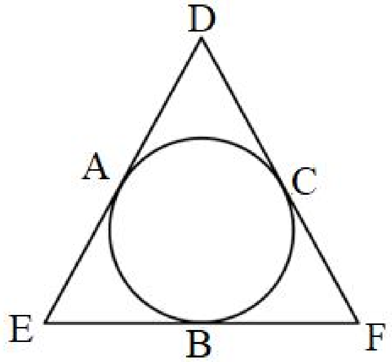
$\therefore \ \alpha+\beta = -\left(\frac{-5}{1}\right) = 5$ and $\alpha\beta = \frac{k}{1} = k$

Now, $\alpha - \beta = 1$ [Given]

So, $(\alpha - \beta)^2 = 1 \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1$

$\Rightarrow 25 - 4k = 1 \Rightarrow 24 = 4k \Rightarrow k = 6$

24. In the given figure, a circle is inscribed in a ΔDEF, such that it touches the sides DE, EF and DF at points A, B and C respectively. If the lengths of sides DE, EF and DF are 9 cm, 13 cm and 11 cm respectively, find the length of BE, CF and AD.



Let BE = EA = x [∵ Tangents drawn from an external point to circle are equal]

$\therefore$ BF = CF = $13 - x$

and DA = DC = $11 - x$

DF = DC + CF = $11 - x + 13 - x = 24 - 2x$

$\Rightarrow 11 = 24 - 2x \Rightarrow 2x = 24 - 11 = 13 \Rightarrow x = \frac{13}{2} = 6.5$

$\therefore$ BE = 6.5 cm, CF = 6.5 cm, AD = 4.5 cm.

25. Draw a line segment PQ of length 7.5 cm. Find a point A and on it such that $\frac{PA}{PQ} = \frac{3}{5}$.

Steps of construction

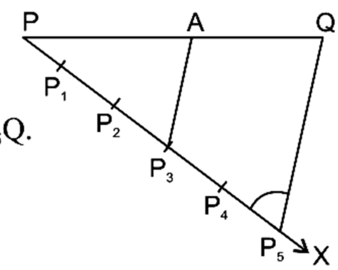
(i) Draw a line segment PQ = 7.5 cm.

(ii) At P, draw an acute angle QPX.

(iii) On PX, draw 5 equal arcs and join P_5Q.

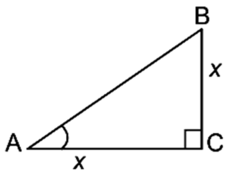
(iv) Draw $\angle PP_5Q = \angle PP_3A$.

Then $\frac{PA}{PQ} = \frac{3}{5}$.



26. In a right triangle ABC, right angled at C, if tan A = 1, then verify that 2sin A cos A = 1.

In Δ ABC, $\tan A = 1 \Rightarrow \frac{BC}{AC} = 1$



BC = x and AC = x

By Pythagoras Theorem,

$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = x^2 + x^2 \Rightarrow AB^2 = 2x^2 \Rightarrow AB = \sqrt{2}\,x$

$\therefore \sin A = \frac{BC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$ and $\cos A = \frac{AC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$

$\therefore 2\sin A \cos A = 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1.$

OR

Evaluate: $2(\cos^2 45° + \tan^2 60°) — 6(\sin^2 45° — \tan^2 30°)$.

$2(\cos^2 45° + \tan^2 60°) - 6(\sin^2 45° - \tan^2 30°)$

$= 2\left\{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\sqrt{3}\right)^2\right\} - 6\left\{\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2\right\}$

$= 2\left(\frac{1}{2} + 3\right) - 6\left(\frac{1}{2} - \frac{1}{3}\right) = 2\left(\frac{1+6}{2}\right) - 6\left(\frac{3-2}{6}\right) = 7 - 1 = 6$

(Question no 27 to 33 are Short Answer Type questions of 3 marks each)

27. Prove that $5 - \sqrt{3}$ is an irrational number, given that $\sqrt{3}$ is irrational.

Let us assume on the contrary that $5 - \sqrt{3}$ is rational. Then, there exits co-prime positive integers a and b, $b \neq 0$ such that

$5 - \sqrt{3} = \frac{a}{b} \Rightarrow 5 - \frac{a}{b} = \sqrt{3}$

$\Rightarrow \frac{5b - a}{b} = \sqrt{3} \Rightarrow \sqrt{3}$ is rational. [$\because$ a, b are integers $\because$ $\frac{5b-a}{b}$ is a rational number]

This contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect.

Hence, $5 - \sqrt{3}$ is an irrational number.

28. The sum of squares of two positive integers is 208. If the square of the larger number is 18 times the smaller number, find the numbers.

Let the smaller number be x. Then, square of larger number = $18x$.

Also, square of the smaller number = x^2.

It is given that the sum of the squares of the integers is 208.

$\therefore x^2 + 18x = 208 \Rightarrow x^2 + 18x - 208 = 0$

$\Rightarrow x^2 + 26x - 8x - 208 = 0 \Rightarrow (x + 26)(x - 8) = 0 \Rightarrow x = 8, \ x = -26$

But, the numbers are positive. Therefore, $x = 8$.

$\therefore$ Square of the larger number = $18x = 18 \times 8 = 144$

Larger number = $\sqrt{144} = 12$

Hence, the numbers are 8 and 12.

OR

Prove that the equation $x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2) = 0$ has no real root, if ad # bc.

Let D be the discriminant of the equation $x^2(a^2 + b^2) + 2x(ac + bd) + (c^2 + d^2) = 0$

Then, $D = 4(ac + bd)^2 - 4(a^2 + b^2)(c^2 + d^2)$

$\Rightarrow D = 4[(ac + bd)^2 - (a^2 + b^2)(c^2 + d^2)]$

$\Rightarrow D = 4[a^2c^2 + b^2d^2 + 2ac.bd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2]$

$\Rightarrow D = 4[2ac.bd - a^2d^2 - b^2c^2] = -4[a^2d^2 + b^2c^2 - 2ad.bc] = -4(ad - bc)^2$

It is given than $ad \neq bc$

$ad - bc \neq 0 \Rightarrow (ad - bc)^2 > 0$

$\Rightarrow -4(ad - bc)^2 < 0 \Rightarrow D < 0$

Hence, the given equation has no real roots.

29. In a circle with centre 0 and radius 5 cm, AB is a chord of length 5 cm. Find the area of the sector AOB. (Take $\pi = 3.14$)

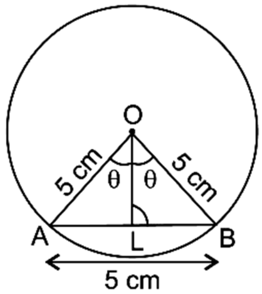
AB = 5 cm

$\Rightarrow AL = BL = \frac{5}{2}$ cm

$\angle AOB = 2\theta$, $\angle AOL = \angle BOL = \theta$

In ΔOLA, $\sin\theta = \frac{AL}{OA} = \frac{\frac{5}{2}}{5} = \frac{1}{2}$

$\Rightarrow \theta = 30^\circ \Rightarrow \angle AOB = 60^\circ$



$\therefore$ Area of sector AOB $= \frac{60}{360} \times \pi \times 5^2 \text{ cm}^2 = 13.1 \text{ cm}^2$

OR

An umbrella has 8 ribs which are equally spaced. Assuming the umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Angle made by two consecutive ribs at the centre $= \frac{360^\circ}{8} = 45^\circ$

Let A be the area between two consecutive ribs

Then, $A = \left(\frac{45}{360} \times \frac{22}{7} \times 45 \times 45\right) \text{ cm}^2$

$\Rightarrow A = \frac{1}{8} \times \frac{22}{7} \times 45 \times 45 \text{ cm}^2 = 795.53 \text{ cm}^2$.

30. Prove that: $\frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} = \sin A + \cos A$

$$\text{LHS} = \frac{\cos A}{1-\tan A} + \frac{\sin A}{1-\cot A} \quad \left[\because \tan A = \frac{\sin A}{\cos A}; \cot A = \frac{\cos A}{\sin A}\right]$$

$$= \frac{\cos A}{1-\frac{\sin A}{\cos A}} + \frac{\sin A}{1-\frac{\cos A}{\sin A}} = \frac{\cos A}{\frac{\cos A - \sin A}{\cos A}} + \frac{\sin A}{\frac{\sin A - \cos A}{\sin A}}$$

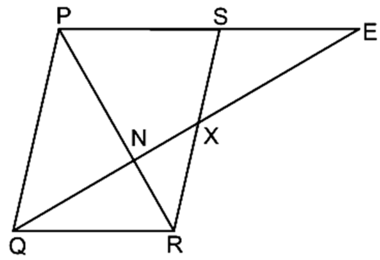
$$= \frac{\cos^2 A}{\cos A - \sin A} + \frac{\sin^2 A}{\sin A - \cos A} = \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A - \sin^2 A}{\cos A - \sin A} = \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A}$$

$= \cos A + \sin A = \text{RHS}$

31. Through the midpoint X of the side RS of a parallelogram PQRS, the line QX is drawn intersecting PR at N and PS produced at E. Prove that EN = 2QN.

In ΔQXR and ΔEXS, we have



$XR = XS$ [$\because$ X is the mid point of RS]

$\angle RXQ = \angle SXE$ [Vertically opposite angles]

and $\angle XRQ = \angle XSE$ [Alternate angles]

$\therefore$ $\Delta QXR \cong \Delta EXS$ [ASA congnence rule]

$\Rightarrow QR = SE$ [CPCT] .... (i)

Also $PS = QR$ [PQRS is a || gm] .... (ii)

$\Rightarrow PS + SE = QR + QR \Rightarrow PE = 2QR$ .... (iii)

Now in ΔPEN and ΔRQN, we have

$\angle PNE = \angle RNQ$ [Vertically opposite angles]

$\Delta EPN \sim \Delta QRN$ [Alternate angles]

$\therefore$ $\Delta PEN \sim \Delta RQN$ [AA similarity criterion]

$$\Rightarrow \frac{EN}{QN} = \frac{PE}{RQ} \Rightarrow \frac{EN}{QN} = \frac{2QR}{QR} \Rightarrow EN = 2QN$$

32. The mean of the following frequency distribution is 25.2. Find the missing frequency x.

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	8	x	10	11	9

Let the missing frequency be f, the assumed mean be A = 47.5 and h = 3.

Class	Frequency (f_i)	Class mark (f_i)	$f_i\, x_1$
0 – 10	8	5	40
10 – 20	x	15	15x
20 – 30	10	25	250
30 – 40	11	35	385
40 – 50	9	45	405
Total	$\sum f_i = 38 + x$		$\sum f_i x_i = 15x + 1080$

Thus, we have, $\sum f_i = 38 + x$; $\sum f_i x_i = 15x + 1080$ and $\bar{x} = 25.2$

We know that, $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$

$$\Rightarrow 25.2 = \frac{5x + 1080}{38 + x} \Rightarrow 25.2(38 + x) = 15x + 1080$$

$$\Rightarrow 957.6 + 25.2x = 15x + 1080 \Rightarrow 25.2x - 15x = 1080 - 957.6$$

$$\Rightarrow 10.2x = 122.4 \Rightarrow x = 12$$ Hence, $x = 12$.

33. The following data gives the distribution of total monthly household expenditure (in rupees) of manual workers in a city:

Expenditure (in Rs.)	1000 – 1500	1500 – 2000	2000 – 2500	2500 – 3000	3000 – 3500	3500 – 4000	4000 – 4500	4500 – 5000
Frequency	24	40	33	28	30	22	16	7

Find the average expenditure which is being done by the maximum number of manual workers.

1500 – 2000 has maximum frequency

∴ It is the modal class such that

$l = 1500$, $h = 500$, $f_1 = 40$, $f_0 = 24$, $f_2 = 33$

$$\therefore \text{ Mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$\text{Mode} = 1500 + \frac{40 - 24}{80 - 24 - 33} \times 500 = 1500 + \frac{16}{23} \times 500 = 1847.826$$

Hence, modal expenditure is ₹1848.

(Question no 34 to 36 are Long Answer Type questions of 5 marks each.)

34. A man on the top of a vertical tower observes a car moving at a uniform speed coming directly towards it. If it takes 12 minutes for the angle of depression to change from 30° to 45°, how soon after this, will the car reach the tower? Give your answer to the nearest second.

Let AB be the tower of height h metres.

Let C be the initial position of the car and

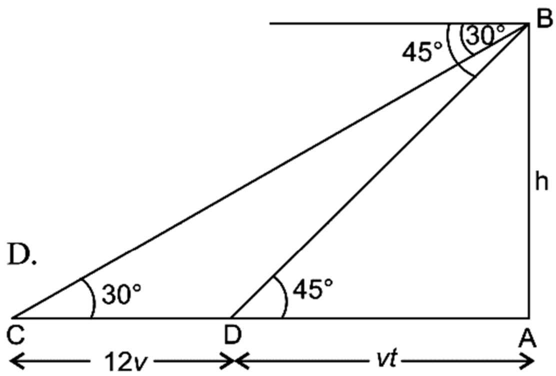
Let after 12 minutes the car be at D.

Let the speed of the car be v metres per minute.

CD = distance travelled by the car in 12 minutes.

⇒ CD = $12v$ metres. Also DA = vt metres,

where t is the time taken to reach the tower AB from D.



In ΔDAB

$$\tan 45° = \frac{AB}{AD} \Rightarrow 1 = \frac{h}{vt} \Rightarrow h = vt \quad \text{... (i)}$$

In ΔCAB, we have

$$\tan 30° = \frac{AB}{AC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{vt + 12v}$$

$$\Rightarrow \sqrt{3}h = vt + 12v \quad \text{... (ii)}$$

From (i) and (ii), we have,

$$\sqrt{3}vt = vt + 12v \Rightarrow \sqrt{3}t = t + 12 \Rightarrow t(\sqrt{3} - 1) = 12$$

$$\Rightarrow t = \frac{12}{\sqrt{3} - 1} \Rightarrow t = \frac{12(\sqrt{3} + 1)}{3 - 1} = 6(\sqrt{3} + 1)$$

$t = 16.39$ minutes or 16 minutes 39 seconds.

OR

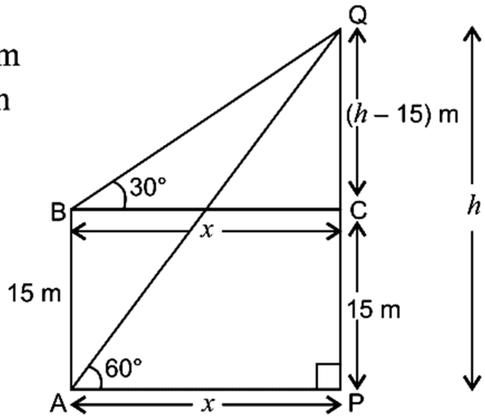
From the top of a building 15 m high, the angle of elevation of the top of a tower is found to be 30°. From the bottom of the same building, the angle of elevation of the top of the tower is found to be 60°. Find the height of the tower and the distance between the tower and the building.

Let AB be the building and PQ be the tower.

Also, $\angle PAQ = 60°$, $\angle CBQ = 30°$ and PC = AB = 15 m

Let BC = AP = x m, PQ = h m, then CQ = $(h - 15)$ m

Now, in ΔAPQ, we have



$$\tan 60° = \frac{PQ}{AP} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}} \quad .... (i)$$

In ΔBCQ, we have, $\tan 30° = \frac{CQ}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h - 15}{x}$

$$\Rightarrow x = \sqrt{3}\ (h - 15) \quad .... (ii)$$

Comparing (i) and (ii), we get, $\frac{n}{\sqrt{3}} = \sqrt{3}(h - 15)$

$$\Rightarrow h = 3(h - 15) \Rightarrow h = 3h - 45 \Rightarrow h - 3h = -45$$

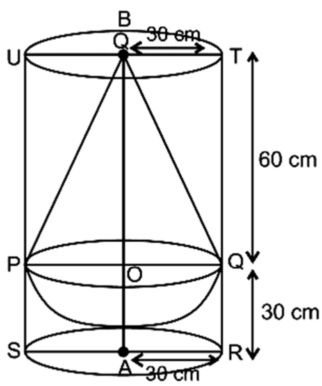
$$\Rightarrow -2h = -45 \Rightarrow h = \frac{45}{2} = 22.5$$

Hence, height of the tower = 22.5 m

Also, distance of the building from the tower $= \frac{h}{\sqrt{3}}$ m $= \frac{22.5}{1.732}$ m = 12.99 metres.

35. A solid consisting of a right cone standing on a hemisphere is placed upright in a right circular cylinder full of water and touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 30 cm and its height is 90 cm, the radius of the hemisphere is 30 cm and height of the cone is 60 cm, assuming that the hemisphere and the cone have common base.

For the cylindrical part, $r = 30$ cm, $h = 90$ cm



$\therefore$ Volume of water that the cylinder contains, $V_1 = \pi r^2 h$

$= \{\pi \times (30)^2 \times 90\}\ cm^3$

For conical part, $r = 30$ cm, $h = 60$ cm

V_2 = volume of the conical part

$= \frac{1}{3}\pi r^2 h = (\frac{1}{3}\pi \times 30^2 \times 60)\ cm^3$

$= (\pi \times 30^2 \times 20)\ cm^3$

For hemispherical part, $r = 30$ cm.

$\therefore$ V_3 = Volume of the hemisphere

$= \left(\frac{2}{3}\pi \times 30^3\right) cm^3 = \pi \times 20 \times 30^2\ cm^3$

Volume of water left in the cylinder

$= (\pi \times 30^2 \times 90 - \pi \times 30^2 \times 20 - \pi \times 20 \times 30^2)\ cm^3$

$= \pi \times 30^2\ (90 - 20 - 20)\ cm^3 = \left(\frac{22}{7} \times 900 \times 50\right)\ cm^3$

$= \frac{22 \times 900 \times 50}{7 \times (100)^3}\ m^3 = \frac{22 \times 45}{7000}\ m^3 = \frac{99}{700}\ m^3 = 0.1414\ m^3.$

36. A train covered a certain distance at a uniform speed. If the train would have been 6 km/hr faster, it would have taken 4 hours less than the scheduled time. And, if the train were slower by 6 km/hr, it would have taken 6 hours more than the scheduled time. Find the length of the journey.

Let the actual speed of the train be x km/hr and the actual time taken be y hours. Then,

Distance covered = xy km.

If the speed is increased by 6 km/hr then the time of journey is reduced by 4 hours, then speed is $(x + 6)$ km/hr, time of Journey is $(y - 4)$ hours.

$\therefore$ Distance covered = $(x + 6)(y - 4)$

$\Rightarrow xy = (x + 6)(y - 4)$

$\Rightarrow -4x + 6y - 24 = 0$

$\Rightarrow -2x + 3y - 12 = 0$ .... (i)

When the speed is reduced by 6 km/hr, then the time of journey is increased by $(y + 6)$ km/hr, i.e. when speed is $(x - 6)$ km/hr, time of journey is $(y - 6)$ hours.

$\therefore$ Distance covered = $(x - 6)(y + 6)$

$\Rightarrow xy = (x - 6)(y + 6)$

$\Rightarrow 6x - 6y - 36 = 0 \Rightarrow x - y - 6 = 0$ .... (ii)

Multiplying (ii) by 2 and adding to (i), we get

$$\begin{array}{r} 2x - 2y - 12 = 0 \\ -2x + 3y - 12 = 0 \\ \hline y = 24 \end{array}$$

Putting $y = 24$ in (ii), we get $x = 30$

Distance = $30 \times 24 = 720$ km

Hence, length of journey = 720 km