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SAMPLE TEST PAPER 05 FOR CLASS X (2020-21)
SAMPLE ANSWER

Max. marks: 80 **Time Allowed: 3 hrs**

General Instruction:
1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part – A:
1. It consists three sections- I and II.
2. Section I has 16 questions of 1 mark each. Internal choice is provided in 5 questions.
3. Section II has 4 questions on case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B:
1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

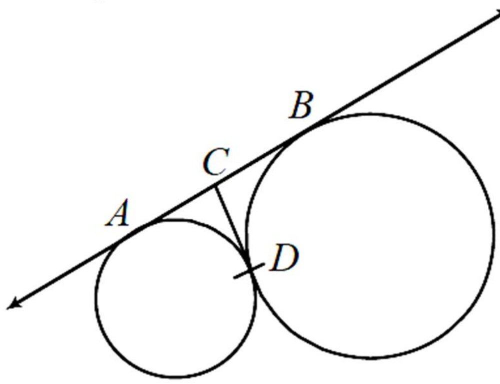
PART - A
SECTION-I
Questions 1 to 16 carry 1 mark each.

1. Find the value(s) of k for which the quadratic equation $2x^2 + kx + 2 = 0$ has equal roots.
Here b = k, a = 2 and c = 2
$k^2 - 4\times2\times2 = 0 \Rightarrow k^2 = 16 \Rightarrow k = \pm4$
Therefore, the values of $k = \pm4$ if roots are equal.

2. Find the number of solutions of the linear equations $\frac{3}{2}x+\frac{5}{3}y=7$ and $9x+10y=14$.

No solution - inconsistent

3. In the below figure, AB and CD are common tangents to circle which touch each other at D. If AB = 8 cm, then find the length of CD.



AC = CD = BC ⇒ CD = 4 cm

4. The first term of an A.P. is 5 and the last term is 45. If the sum of all the terms is 400, find the number of terms.
Number of terms = 16

OR

Find the 9th term of the A.P. – 15, –11, –7, ..., 49
9th term = 17

5. Form a quadratic polynomial, the sum and product of whose zeros are (–3) and 2 respectively.
Ans: $x^2 + 3x + 2$

6. If x = p, y = q is a solution of the equations x + 2y + 1 = 0 and 2x – 3y – 12 = 0, then find the values of p and q.

Since $x = p$, $y = q$ is a solution of the equations

$p + 2q + 1 = 0 \Rightarrow p + 2q = -1$ ...(i)

$2p - 3q - 12 = 0 \Rightarrow 2p - 3q = 12$ ...(ii)

Multiplying (i) by 2, and then subtracting it from (ii), we get

$4q + 3q = -14 \Rightarrow q = -2$

Putting $q = -2$ in (i), we get

$p = -14 + 4 = 3$

So, $p = 3$, $q = -2$

7. Given that HCF (135, 225) = 45, find the LCM (135, 225).

$$\text{LCM} = \frac{135 \times 225}{45} = 675$$

OR

After how many decimal places will the decimal representation of the rational number $\frac{229}{2^2 \times 5^7}$ terminate?

After 7 decimal places

8. Find the value of k for which the quadratic equation $x^2 - 4x + k = 0$ has distinct real roots.

$x^2 - 4x + k = 0$

$D = b^2 - 4ac = (-4)^2 - 4(1)(k) = 16 - 4k$

The roots are real and distinct: $D > 0$

$\Rightarrow 16 - 4k > 0 \Rightarrow -4k > -16 \Rightarrow 4k < 16 \Rightarrow k < 4$

OR

Find the roots of the equation $6x^2 + 11x + 3 = 0$.

$6x^2 + 11x + 3 = 0$

$\Rightarrow 6x^2 + 9x + 2x + 3 = 0$

$\Rightarrow (2x + 3)(3x + 1) = 0$

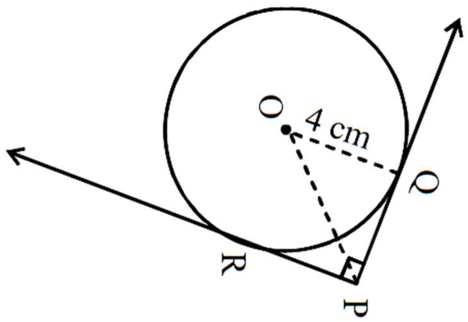
$\Rightarrow x = -3/2, x = -1/3$

9. If tan A = 1, then find the value of 2 sin A cos A.

$\tan A = 1 \Rightarrow A = 45°$

$2\sin A \cos A = 2\sin 45° \cos 45° = 2(1/\sqrt{2})(1/\sqrt{2}) = 1$

10. In the below figure, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If ∠QPR = 90°, then length of PQ



Since tangent is perpendicular to the radius through the point of contact, therefore OQ is perpendicular to QP,

Similarly, OR is perpendicular to PR.

Now, OR = OQ= 4 cm ....(radius of the circle)

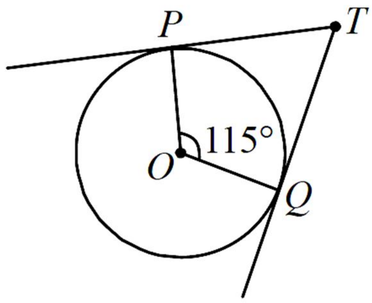
Now, consider the tangents PQ and PR, we get,
PQ = PR........(tangent from a same point that is P)

Also, $\angle$QPR = 90

So, PQOR is a square.
So since it is a square, so all the sides are same,
OQ = OR = RP = QP = 4 cm

OR

In the below figure, TP and TQ are tangents drawn to the circle with centre at O. If ∠POQ = 115° then find ∠PTQ.



Here, ∠PTQ = 65°, ∠P = 90° and ∠Q=90° , ∠O = 115°
In quadrilateral OPTQ ,
sum of opposite angles of a quadrilateral is 180°
∴ ∠O +∠T = 180°
⇒ 115 + ∠T = 180 °
⇒∠ T = 180 - 115 = 65°

11. If the diameter of semicircular protractor is 14cm, find its perimeter.

Perimeter of the protractor = πr + diameter = $\frac{22}{7} \times 7$ cm + 14 cm = 36 cm

12. Find the radius of a sphere (in cm) whose volume is 12π cm^3.
Volume of Sphere = $4/3\ \pi\ r^3$
Here volume is $12\ \pi$ cm^3
∴ $4/3\ \pi\ r^3 = 12\ \pi$ cm^3 ⇒ $4 r^3 = 36$ ⇒ $r^3 = 9$ ⇒ $r = 3^{2/3}$

13. Let ΔABC ~ ΔDEF and their areas be respectively 81 cm^2 and 144 cm^2. If EF = 24 cm, then find the length of side BC.
BC = 18 cm

14. To draw a pair of tangents to a circle which are inclined to each other at an angle x^o, what is the angle to be constructed between the two radii which are drawn at the points of contact?
The angle to be constructed is $(180 - x°)$

15. If $\sqrt{3}\sin\theta - \cos\theta = 0$ and $0^0 < \theta < 90^0$, find the value of θ.

$\sqrt{3}\sin\theta - \cos\theta = 0 \Rightarrow \cos\theta = \sqrt{3}\sin\theta$

$\Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} = \tan 30° \Rightarrow \theta = 30°.$

16. Two dice are thrown simultaneously. What is the probability that the sum of the two numbers appearing on the top is 13?
Probability that the sum of the two numbers appearing on the top is 13 = 0

OR

A letter is chosen at random from the letters of the word "RHYTHM". Find the probability that the letter chosen is a vowel.

No of vowels in “RHYTHM” = 0

P(a vowel) = 0

SECTION-II

Case study based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. Case Study based-1: Safety Board

Aditya is celebrating his birthday. He invited his friends. He bought a packet of toffees/candies which contains 120 candies. He arranges the candies such that in the first row there are 3 candies, in second there are 5 candies, in third there are 7 candies and so on.

(a) Find the total number of rows of candies.

(i) 12 (ii) 10 (iii) 14 (iv) 8

Ans: (ii) 10

(b) How many candies are placed in last row?

(i) 22 (ii) 21 (iii) 24 (iv) 18

Ans: (ii) 21

(c) Find the difference in number of candies placed in 7th and 3rd row.

(i) 8 (ii) 10 (iii) 12 (iv) 14

Ans: (i) 8

(d) If Aditya decides to make 15 rows, then how many total candies will be placed by him with the same arrangement?

(i) 200 (ii) 150 (iii) 255 (iv) 210

Ans: (iii) 255

(e) Find the number of candies in 12th row.

(i) 21 (ii) 30 (iii) 25 (iv) 19

Ans: (iii) 25

18. Case Study based-2:

Ruby and Rita are best friends. They are staying in the same colony. Both are studying in the same class and in the same school. During Winter vacation Ruby visited Rita’s house to play Ludo. They decided to play Ludo with 2 dice.

(a) To win a game, Ruby wanted a total of 7.
What is the probability of winning a game by Ruby?

(i) $\frac{1}{6}$ (ii) $\frac{7}{12}$ (iii) $\frac{5}{18}$ (iv) $\frac{1}{9}$

Ans: (i) $\frac{1}{6}$

(b) To win a game, Rita wanted 8 as the sum. What is the probability of winning a game by Rita?

(i) $\frac{1}{12}$ (ii) $\frac{7}{36}$ (iii) $\frac{5}{36}$ (iv) $\frac{1}{4}$

Ans: (iii) $\frac{5}{36}$

(c) What is the probability that the sum of the numbers on the both the dice is divisible by 4 or 6?

(i) $\frac{7}{18}$ (ii) $\frac{7}{15}$ (iii) $\frac{5}{18}$ (iv) $\frac{2}{9}$

Ans: (i) $\frac{7}{18}$

(d) The probability of getting a total of atleast 10 is:

(i) $\frac{1}{6}$ (ii) $\frac{1}{3}$ (iii) $\frac{2}{3}$ (iv) $\frac{1}{4}$

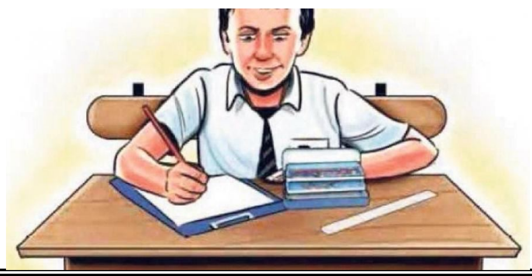
Ans: (i) $\frac{1}{6}$

(e) The probability that 5 will come up at least in 1 die is:

(i) $\frac{7}{36}$ (ii) $\frac{11}{36}$ (iii) $\frac{25}{36}$ (iv) $\frac{2}{9}$

Ans: (ii) $\frac{11}{36}$

19. Case Study based-3:
In a school, Class X B and C students appeared for Sunday Sample paper test 05 and marks obtained out of 80 are formulated in a table as follows:



Marks	Number of students
Less than 10	8
Less than 20	20
Less than 30	30
Less than 40	50
Less than 50	60
Less than 60	70
Less than 70	75
Less than 80	80

(a) How many students secured less than 40 marks?
(i) 50 (ii) 40 (iii) 60 (iv) 30
Ans: (i) 50

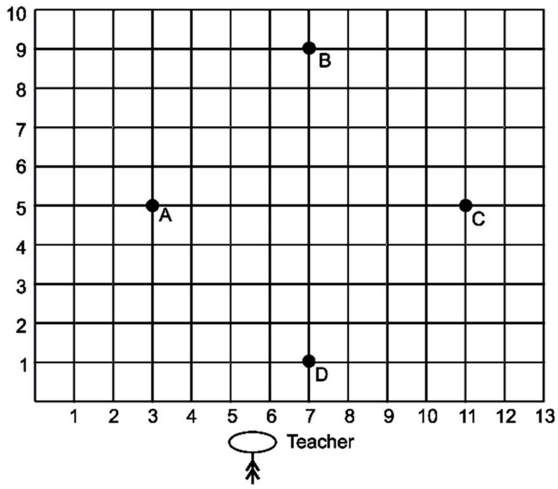
(b) What is the upper limit of modal class?
(i) 20 (ii) 30 (iii) 40 (iv) 50
Ans: (iii) 40

(c) The median class is :
(i) 10-20 (ii) 20-30 (iii) 30-40 (iv) 40-50
Ans: (iii) 30-40

(d) The mean marks of the students is :
(i) 35.8 (ii) 35.9 (iii) 36 (iv) 36.5
Ans: (ii) 35.9

(e) Class mark of the class preceding the modal class is :
(i) 35 (ii) 30 (iii) 25 (iv) 45
Ans: (iii) 25

20. Case Study based-3:
Students of a school are standing in rows and columns in their playground for a drill practice. A, B, C and D are the positions of four students as shown in the figure.



Answer the following questions.
(a) What are the coordinates of A and B respectively?
(i) A(3, 5); B(7, 8) (ii) A(5, 3); B(8, 7) (iii) A(3, 5); B(7, 9) (iv) A(5, 3); B(9, 7)
Sol. (iii) A(3, 5); B(7, 9)

(b) What are the coordinates of C and D respectively?
(i) C(11, 5); D(7, 1) (ii) C(5, 11); D(1, 7) (iii) C(5, 11); D(7, 1)(iv) C(5, 11); D(-1, 7)
Sol. (i) C(11, 5); D(7, 1)

(c) Is it possible to place Ram(R) in the drill in such a way that he is equidistant from all the four students A, B, C and D?
(i) yes (ii) not possible (iii) not sure (iv) none
Sol. (i) yes

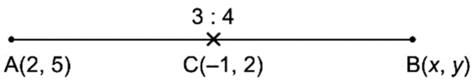
(d) What are the coordinates of the position of Ram?
(i) (7, 5) (ii) (5, 7) (iii) (7, 7) (iv) (5, 5)
Sol. (i) (7, 5)

(e) What is the distance between B and D?
(i) 5 units (ii) 14 units
(iii) 8 units (iv) 10 units
Sol. (iii) 8 units

PART – B

(Question No 21 to 26 are Very short answer Type questions of 2 mark each)

21. If the point C(-1, 2) divides internally the line segment joining A(2, 5) and B(x, y) in the ratio 3 : 4, find the coordinates of B.



C divides join of A (2, 5) and B (x, y) in the ratio 3 : 4

$\therefore$ Coordinates of C are $\left(\frac{3x+8}{3+4}, \frac{3y+20}{3+4}\right) = (-1, 2)$

$$\Rightarrow \left(\frac{3x+8}{7}, \frac{3y+20}{7}\right) = (-1, 2) \Rightarrow \frac{3x+8}{7} = -1, \frac{3y+20}{7} = 2$$

$$\Rightarrow 3x + 8 = -7, 3y + 20 = 14 \Rightarrow 3x = -15, 3y = -6$$

$$\Rightarrow x = -5, y = -2$$

$\therefore$ Coordinates of B are (–5, –2)

OR

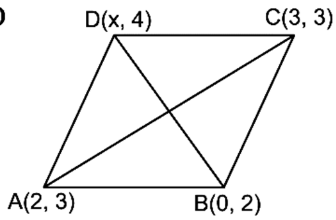
If A(2, 3), B(0, 2), C(3, 3) and D(x, 4) are the vertices of a ||gm ABCD, then what is the value of x?

Since diagonals of a ||gm bisect each other, so,

mid-point of AC = mid-point of BD

$$\Rightarrow \left(\frac{2+3}{2}, \frac{3+3}{2}\right) = \left(\frac{x+0}{2}, \frac{4+2}{2}\right)$$

$$\Rightarrow \left(\frac{5}{2}, 3\right) = \left(\frac{x}{2}, 3\right) \Rightarrow x = 5$$



22. Find the greatest possible speed (in km/hr) at which a bird should fly to cover a distance of 45 km and 336 km in exact number of hours.

The greatest possible speed = HCF of 45, 336

$45 = 3^2 \times 5$, $336 = 2^4 \times 3 \times 7$

So, HCF (45, 336) = 3

So, required speed = 3 km/h.

23. If $\sin A + \sin^2 A = 1$, then find $\cos^2 A + \cos^4 A$.

Given that $\sin A + \sin^2 A = 1$

$\Rightarrow \sin A = 1 - \sin^2 A \quad [\because \sin^2 A + \cos^2 A = 1]$

$= \cos^2 A$

$\therefore \sin^2 A = \cos^4 A$

Now $\cos^2 A + \cos^4 A = \sin A + \sin^2 A = 1$

OR

Simplify: $\sqrt{\frac{1-\sin^2 A}{\tan^2 A+1}}$

$$\sqrt{\frac{1-\sin^2 A}{\tan^2 A+1}} = \sqrt{\frac{\cos^2 A}{\sec^2 A}} = \frac{\cos A}{\sec A} = \cos A \times \cos A = \cos^2 A$$

24. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c$, $a \neq 0$, $c \neq 0$.

Given polynomial $f(x) = ax^2 + bx + c, a \neq 0, c \neq 0$

Let zeroes be α, β

$$\therefore \quad \alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a} \quad ...(i)$$

For required polynomial

Reciprocal zeroes are $\frac{1}{\alpha}, \frac{1}{\beta}$

Sum $= \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{-b/a}{c/a} = -\frac{b}{c} = -\frac{B}{A}$

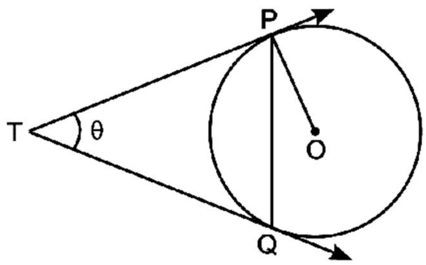
Product $= \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{c/a} = \frac{a}{c} = \frac{C}{A}$

Polynomial is $Ax^2 + Bx + C$

Here, $A = c$, $B = b$, $C = a$

i.e. $cx^2 + bx + a$

25. In the below figure, two tangents TP and TQ are drawn to a circle with centre 0 from an external point T. Prove that ∠PTQ = 2∠OPQ.



Let $\angle PTQ = \theta$

Now $TP = TQ$ [Lengths of tangent segments from an external point to a circle are equal]

$\therefore$ ΔTPQ is an isosceles triangle

$\Rightarrow$ $\angle TPQ = \angle TQP$ [Angles opposite to equal sides of a triangle are equal]

In triangle TPQ $\angle PTQ + \angle TPQ + \angle TQP = 180°$

$\Rightarrow$ $\theta + 2\angle TPQ = 180°$ [Angle sum property of triangles]

$\Rightarrow$ $\angle TPQ = \frac{1}{2}(180° - \theta) = 90° - \frac{1}{2}\theta$

Also, $\angle OPT = 90°$ [Angle between tangent and radius through the point of contact]

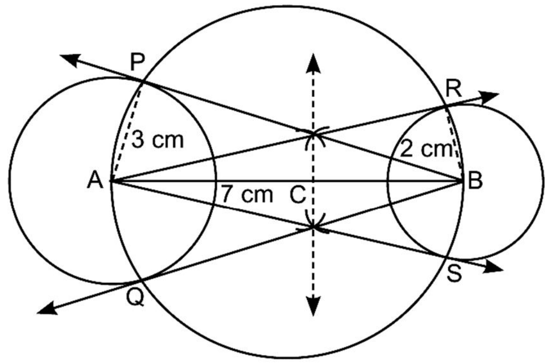
So, $\angle OPQ = \angle OPT - \angle TPQ = 90° - \left(90° - \frac{1}{2}\theta\right) = \frac{1}{2}\theta \Rightarrow \angle OPQ = \frac{1}{2}\angle PTQ$

or $\angle PTQ = 2\angle OPQ$

26. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Steps of construction:

1. Draw AB = 7 cm. Taking A and B as centres, draw two circles of 3 cm and 2 cm radius.
2. Bisect line AB. Let mid-point of AB be C.
3. Taking C as centre, draw circle of AC radius which will intersect circles at P, Q, R, S. Join BP, BQ, AR, AS.
4. Required tangents are (*i*) BP and BQ (*ii*) AR and AS.



(Question no 27 to 33 are Short Answer Type questions of 3 marks each)

27. Prove that $2 - \sqrt{7}$ is an irrational number, given that $\sqrt{7}$ is irrational.

Let us assume that $2-\sqrt{7}$ is rational.

$\therefore$ $2-\sqrt{7} = \frac{a}{b}$, where a and b are integers and $b \neq 0$,

$\therefore$ $\sqrt{7} = 2 - \frac{a}{b} = \frac{2b-a}{b}$

Since, a and b are integers, $\frac{2b-a}{b}$ is rational. So, $\sqrt{7}$ is rational. But we know that $\sqrt{7}$ is irrational, which is a contradiction as the RHS is rational. This contradiction has arised due to our wrong assumption.

$\Rightarrow$ Our assumption is wrong

$\therefore$ $2-\sqrt{7}$ is irrational.

28. In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/hr and time of flight increased by 30 minutes. Find the original duration of flight.

Let original speed of the aircraft be x km/hr

Reduced speed = $(x - 200)$ km/hr

According to given condition

$$\frac{600}{x-200} - \frac{600}{x} = \frac{30}{60} = \frac{1}{2} \Rightarrow \frac{600x - 600x + 120000}{x(x-200)} = \frac{1}{2}$$

$$\Rightarrow \frac{120000}{x^2 - 200x} = \frac{1}{2} \Rightarrow x^2 - 200x = 240000 \Rightarrow x^2 - 200x - 240000 = 0$$

$$\Rightarrow x^2 - 600x + 400x - 240000 = 0 \Rightarrow x(x-600) + 400(x-600) = 0$$

$$\Rightarrow (x+400)(x-600) = 0 \Rightarrow x + 400 = 0 \text{ or } x - 600 = 0$$

$$\Rightarrow x = -400 \text{ (rejected) or } x = 600$$

$\therefore$ original speed = 600 km/hr

$\therefore$ original duration of flight = $\frac{600}{600}$ = 1 hour

OR

A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

Let original speed of train be x km/hr

Now, new speed be = $(x - 8)$ km/hr

$$\therefore \frac{480}{x-8} - \frac{480}{x} = 3 \Rightarrow \frac{480x - 480x + 3840}{x(x-8)} = 3$$

$$\Rightarrow 3x(x-8) = 3840 \Rightarrow x^2 - 8x - 1280 = 0$$

$$\Rightarrow x^2 - 40x + 32x - 1280 = 0$$

$$\Rightarrow x(x-40) + 32(x-40) = 0$$

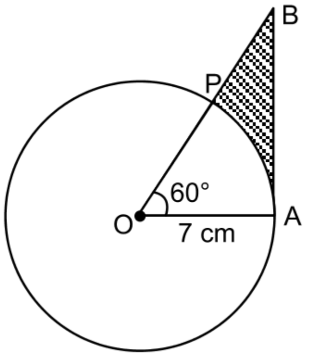
$$\Rightarrow (x+32)(x-40) = 0$$

$$\Rightarrow x + 32 = 0 \text{ or } x - 40 = 0$$

$$\Rightarrow x = -32 \text{(rejected) or } x = 40$$

$\therefore$ original speed of train = 40 km/hr

29. Find the perimeter and area of the shaded region in the below figure.



Since tangent AB is perpendicular to radius OA, so $\angle OAB = 90°$

Now, $\tan 60° = \frac{AB}{OA} \Rightarrow \sqrt{3} = \frac{AB}{7} \Rightarrow AB = 7\sqrt{3}$

Now, $\cos 60° = \frac{OA}{OB} \Rightarrow \frac{1}{2} = \frac{7}{OB}$

$\therefore$ OB = 14 cm

PB = OB – OP = (14 – 7) cm = 7 cm

$\Rightarrow \widehat{AP} = \frac{\pi r\theta}{180°} = \frac{22}{7} \times \frac{7 \times 60°}{180°} = \frac{22}{3}$ cm.

Now perimeter of shaded part = PB + $\widehat{AP}$ + AB

$= 7 + \frac{22}{3} + 7\sqrt{3} = \frac{21 + 21\sqrt{3} + 22}{3} = \left(\frac{43 + 21\sqrt{3}}{3}\right)$ cm

Area of the shaded region = ar (ΔOAB) – ar (sector OPA)

$= \frac{1}{2} \times 7\sqrt{3} \times 7 - \frac{22}{7} \times \frac{7 \times 7 \times 60}{360} = \frac{49\sqrt{3}}{2} - \frac{154}{6} = \frac{7}{2}\left(7\sqrt{3} - \frac{22}{3}\right) \text{cm}^2$

30. Prove that $2(\sin^6\theta + \cos^6\theta) — 3(\sin^4\theta + \cos^4\theta) + 1 = 0$.

Consider $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2\{(\sin^2\theta)^3 + (\cos^2\theta)^3\} - 3\{(\sin^2\theta)^2 + (\cos^2\theta)^2\} + 1$

Using $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$ and $a^2 + b^2 = (a+b)^2 - 2ab$, we get

$= 2[\{\sin^2\theta + \cos^2\theta\}^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)\}]$

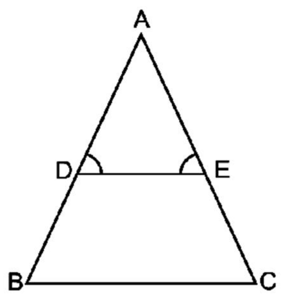
$-3[\{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta\}] + 1$

$= 2[(1)^3 - 3\sin^2\theta\cos^2\theta(1)] - 3[(1)^2 - 2\sin^2\theta\cos^2\theta] + 1$

$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1$

$= 0$

31. In the below figure, $\angle D = \angle E$ and $\frac{AD}{DB} = \frac{AE}{EC}$, prove that BAC is an isosceles triangle.



In ΔADE, $\angle D = \angle E$ (given)

$\Rightarrow$ AE = AD ...(*i*) (sides opposites to equal angles)

Also, $\frac{AD}{DB} = \frac{AE}{EC}$ (given)

$\Rightarrow$ DB = EC

From (*i*) and (*ii*) AD + DB = AE + EC

$\Rightarrow$ AB = AC

$\Rightarrow$ BAC is an isosceles triangle.

32. Find the mean of the following distribution:

Class	3 – 5	5 – 7	7 – 9	9 – 11	11 – 15
Frequency	5	10	10	7	8

Table for mean: let $A = 8$, here $h = 2$

C.I.	x	f	$d = \frac{x-8}{2}$	fd	fx
3 – 5	4	5	–2	–10	20
5 – 7	6	10	–1	–10	60
7 – 9	8	10	0	0	80
9 – 11	10	7	1	7	70
11 – 13	12	8	2	16	96
		40		3	326

$$\text{Mean} = A + \frac{\Sigma fd}{\Sigma f} \times h = 8 + \frac{3}{40} \times 2 = 8 + 0.15 = 8.15$$

Alt. method: $$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{326}{40} = 8.15$$

OR

Find the mode of the following distribution:

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140
Frequency	6	8	10	12	6	5	3

Table for mode:

C.I.	f
0 – 20	6
20 – 40	8
40 – 60	10
60 – 80	12
80 – 100	6
100 – 120	5
120 – 140	3

Maximum frequency (f_0) = 12

Modal class is 60 – 80

$f_0 = 12, f_1 = 10, f_2 = 6, h = 20, l = 60$

← Modal class

$$\therefore \text{ Mode} = l + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$$

$$= 60 + \frac{12-10}{24-10-6} \times 20 = 60 + \frac{2}{8} \times 20 = 65$$

33. The median of the following data is 525. Find the values of x and y, if total frequency is 100:

Class	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600	600 – 700	700 – 800	800 – 900	900 – 1000
Frequency	2	5	x	12	17	20	y	9	7	4

Table for median = 525

C.I.	f	$c.f$
0–100	2	2
100–200	5	7
200–300	x	$7 + x$
300–400	12	$19 + x$
400–500	17	$36 + x$
500–600	20	$56 + x$ ← Median class
600–700	y	$56 + x + y$
700–800	9	$65 + x + y$
800–900	7	$72 + x + y$
900–1000	4	$76 + x + y$
	100	

We have $76 + x + y = 100 \Rightarrow x + y = 24$ ...(i)

As median is 525, therefore median class is 500 – 600.

$l = 500, \frac{N}{2} = \frac{100}{2} = 50, c = 36 + x, f = 20, h = 100$

$$\text{Median} = l + \frac{\frac{N}{2} - c}{f} \times h$$

$$525 = 500 + \frac{50 - (36 + x)}{20} \times 100$$

$$25 = (50 - 36 - x)5$$

$$25 = 70 - 5x$$

$\Rightarrow 5x = 45 \Rightarrow x = 9$

from (i) $9 + y = 24 \Rightarrow y = 15$

$\therefore x = 9, y = 15$

(Question no 34 to 36 are Long Answer Type questions of 5 marks each.)

34. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Let height of the tower AB be 'H' m

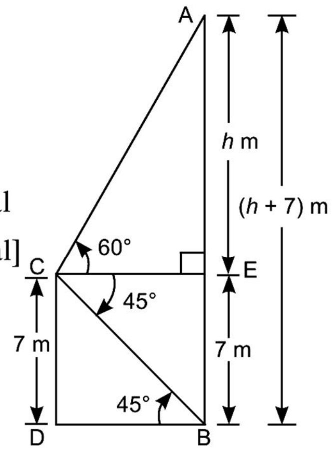
$\therefore$ H = AE + EB $\Rightarrow$ H = (h + 7)m

Given height of building CD = 7 m

$\angle$ACE = 60°, $\angle$ECB = 45°

$\Rightarrow$ $\angle$CBD = 45° [$\because$ CE || DB, CB is transversal

$\therefore$ alternate pair of angles are equal]



Now in right-triangle CDB,

$$\frac{CD}{DB} = \tan 45° \Rightarrow \frac{7}{DB} = 1$$

$\Rightarrow$ DB = 7 m

In right triangle AEC,

$$\frac{AE}{EC} = \tan 60°$$

$\Rightarrow \frac{h}{7} = \sqrt{3}$ [$\because$ DB = EC = 7 m]

$\Rightarrow h = 7\sqrt{3}$ m

Now, H = AB = $h + 7 = 7\sqrt{3} + 7 = 7(\sqrt{3} + 1)$ m

Hence, height of the tower is $7(\sqrt{3} + 1)$m

OR

A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. (Take $\sqrt{3}$ = 1.73)

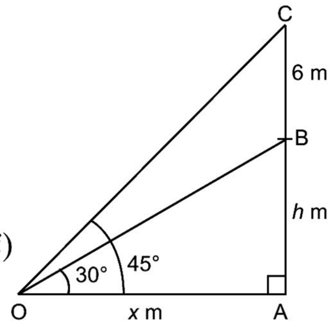
AB → Tower of height h m

BC → flag slide of height 6 m.

O is point of observation, ∠AOB = 30°, ∠AOC = 45°

Let OA = x m

In right-angled triangle OAB.



$$\frac{AB}{OA} = \tan 30° \Rightarrow \frac{h}{x} = \frac{1}{\sqrt{3}} \Rightarrow x = \sqrt{3}h \quad ...(i)$$

In right-angled triangle OAC $\frac{AC}{OA} = \tan 45°$

$$\Rightarrow \frac{h+6}{x} = 1 \Rightarrow h + 6 = x \Rightarrow h + 6 = \sqrt{3}h \quad \text{[from (}i\text{)]}$$

$$\Rightarrow \sqrt{3}h - h = 6 \Rightarrow (\sqrt{3} - 1)h = 6$$

$$\Rightarrow h = \frac{6}{\sqrt{3}-1} = \frac{6(\sqrt{3}+1)}{3-1} = 3(\sqrt{3} + 1)$$

$$\Rightarrow h = 3(1.73 + 1) = 3 \times 2.73 = 8.19 \text{ m}$$

∴ height of tower = 8.19 m.

35. Water is flowing through a cylindrical pipe of internal diameter 2 cm, into a cylindrical tank of base radius 40 cm at the rate of 0.7 m/sec. By how much will the water rise in the tank in half an hour?

For pipe, r = 1 cm

Length of water flowing in 1 sec, h = 0.7 m = 70 cm

Cylindrical tank, R = 40 cm, rise in water level = H

Volume of water flowing in 1 sec = $\pi r^2 h = \pi \times 1 \times 1 \times 70$ cm^3 = 70π cm^3

Volume of water flowing in 60 sec = $70\pi \times 60$ cm^3

Volume of water flowing in 30 minutes =$70\pi \times 60 \times 30$ cm^3

Volume of water in tank = $\pi R^2 H = \pi \times 40 \times 40 \times H$

Volume of water in tank = Volume of water flowing in 30 minutes

$\Rightarrow \pi \times 40 \times 40 \times H = 70\pi \times 60 \times 30 \Rightarrow H = 78.75$ cm.

Hence, the water level will rise by 30 minutes.

36. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by 2y – x = 8, 5y – x = 14 and y – 2x = 1.

Consider equation $2y - x = 8$

$\Rightarrow \; x = 2y - 8$

Some points on graph are

x	0	–8	–4
y	4	0	2

Consider equation $5y - x = 14$

$\Rightarrow \; x = 5y - 14$

Some points on graph are

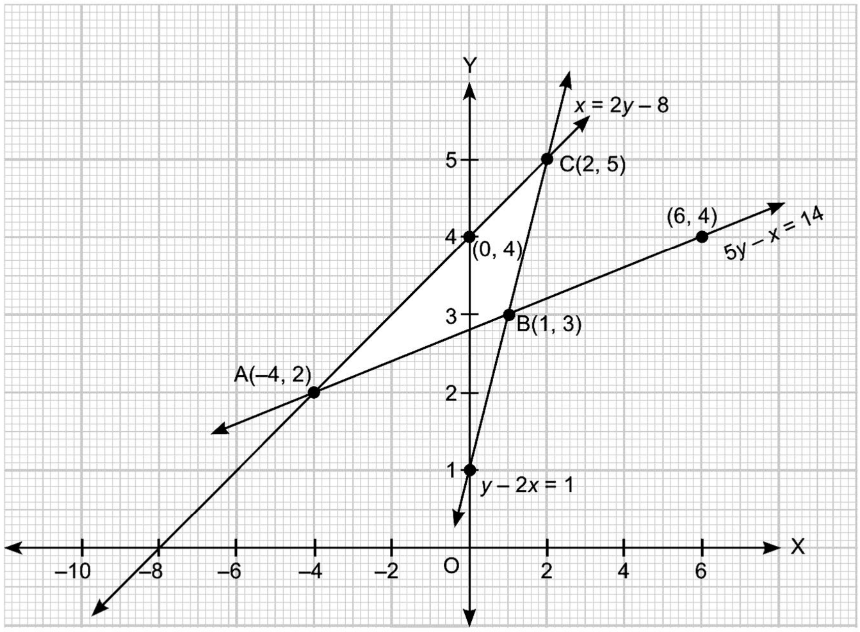
x	–4	1	6
y	2	3	4

Consider equation $y - 2x = 1$

$\Rightarrow \; y = 2x + 1$

Some points on graph are

x	0	1	2
y	1	3	5



Plotting the points on graph we get triangle ABC with vertices A(–4. 2), B(1, 3), C(2, 5)