KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS HYD – 32
SAMPLE TEST PAPER 07 FOR CLASS X BOARD EXAM 2021
(SAMPLE ANSWERS)

Max. marks: 80 **Time Allowed: 3 hrs**

General Instruction:
1. This question paper contains two parts A and B.
2. Both Part A and Part B have internal choices.

Part – A:
1. It consists three sections- I and II.
2. Section I has 16 questions of 1 mark each. Internal choice is provided in 5 questions.
3. Section II has 4 questions on case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B:
1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.
4. Internal choice is provided in 2 questions of 2 marks, 2 questions of 3 marks and 1 question of 5 marks.

PART - A
SECTION-I
Questions 1 to 16 carry 1 mark each.

1. Find the values of k for which the quadratic equation $2kx^2 – 40x + 25 = 0$ has equal roots.

The given equation is of the form $ax^2 + bx + c = 0$

where $a = 2k$, $b = -40$ and $c = 25$.

$D = b^2 - 4ac = (-40)^2 - 4 \times 2k \times 25 = 1600 - 200k$

The equation will have equal roots, if

$D = 0 \quad \Rightarrow 1600 - 200k = 0 \quad \Rightarrow 200k = 1600 \quad \Rightarrow \; k = 8.$

2. Determine the number of solutions of the pair of linear equations:
$2x – 3y – 5 = 0$, $6y – 4x – 3 = 0$.

The given equations are of the form

$a_1 x + b_1 y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$

where $a_1 = 2, b_1 = -3, c_1 = -5$ and $a_2 = 6, b_2 = -4, c_2 = -3$

$\therefore \; \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-3}{-4} = \frac{3}{4}, \quad \frac{c_1}{c_2} = \frac{-5}{-3} = \frac{5}{3}$

Thus, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Hence, the given system of equations is consistent and has a unique solution.

3. Find the edge of the cube whose total surface area is 433.5 cm².

Let the edge of the cube be a cm.

Then, its total surface area = $6a^{2.}$

$\therefore \quad 6a^2 = 433.5 \quad \Rightarrow a^2 = 72.25 \quad \Rightarrow a = 8.5$

Hence, the edge of the cube is 8.5 cm.

4. Find the numerical value of $\cos^2\theta + \dfrac{1}{1+\cot^2\theta}$.

$$\cos^2\theta + \frac{1}{1+\cot^2\theta} = \cos^2\theta + \frac{1}{\operatorname{cosec}^2\theta} \quad [\because \operatorname{cosec}^2\theta - \cot^2\theta = 1]$$

$$= \cos^2\theta + \sin^2\theta \qquad \left[\because \sin\theta = \frac{1}{\operatorname{cosec}\theta}\right]$$

$$= 1$$

OR

Find the value of ($\operatorname{cosec}^2 60° + \tan^2 30° + \cot^2 60°$).

$\operatorname{cosec}^2 60° + \tan^2 30° + \cot^2 60°$

$$= \left(\frac{2}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{4}{3} + \frac{1}{3} + \frac{1}{3} = \frac{6}{3} = 2.$$

5. Find the distance between the points (0, sec θ) and (–tan θ, 0).

$$\text{Required distance} = \sqrt{(-\tan\theta - 0)^2 + (0 - \sec\theta)^2}$$

$$= \sqrt{\tan^2\theta + \sec^2\theta} = \sqrt{\frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta}}$$

$$= \sqrt{\frac{\sin^2\theta + 1}{\cos^2\theta}} = \frac{\sqrt{\sin^2\theta + 1}}{\cos\theta}$$

6. Find the common difference of an AP whose 6th term is 12 and the 8th term is 22.

Let a and d be the first term and the common difference of the given AP. Then,

$T_6 = 12 \quad \Rightarrow a + 5d = 12 \qquad ...(i)$

$T_8 = 22 \quad \Rightarrow a + 7d = 22 \qquad ...(ii)$

On subtracting (*i*) from (*ii*), we get:

$2d = 10 \quad \Rightarrow d = 5.$

Hence, the common difference of the given AP is 5.

7. Find the product of the zeros of the polynomial $4x^3 - 7x^2 + 3x - 2$.

Let α, β and γ be the zeros of the polynomial $4x^3 - 7x^2 + 3x - 2$.

$$\text{Then, } \alpha\beta\gamma = -\left(\frac{\text{Constant term}}{\text{Coefficient of } x^3}\right) = -\left(\frac{-2}{4}\right) = \frac{1}{2}.$$

OR

Find a quadratic polynomial, the sum and product of whose zeros are 7/2 and −5/4 respectively.

Let α and β be the zeros of the required polynomial.

Then, $\alpha + \beta = \dfrac{7}{2}$ and $\alpha\beta = \dfrac{-5}{4}$.

Now, for suitable value of k the polynomial is given by

$k\,(x^2 - (\alpha+\beta)\,x + \alpha\beta)$

$$\Rightarrow k\left\{x^2 - \frac{7}{2}x + \left(\frac{-5}{4}\right)\right\} \Rightarrow \frac{k}{4}\,(4x^2 - 14x - 5) \Rightarrow 4x^2 - 14x - 5 \text{ for } k = 4.$$

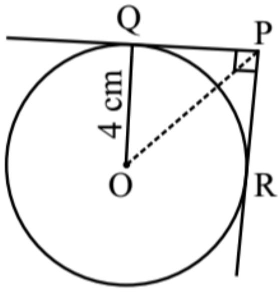
8. If the point (x, 4) lies on a circle whose centre is at the origin and radius is 5, then find the value of x.

Let $O(0, 0)$ be the centre of a circle and let $A(x, 4)$ be a point on this circle.

Then $OA = 5 \quad \Rightarrow OA^2 = 25 \quad \Rightarrow (x - 0)^2 + (4 - 0)^2 = 25$

$\Rightarrow x^2 + 16 = 25 \quad \Rightarrow x^2 = 9 \quad \Rightarrow x = \pm 3.$

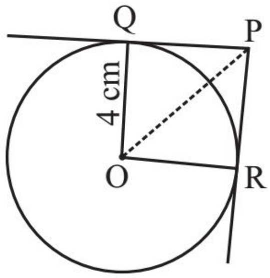
9. In the given below figure, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If ∠QPR = 90°, then find the length of PQ.



Join OR.

Then, $\angle OQP = \angle ORP = 90°$

[∵ Radius through the point of contact is perpendicular to the tangent]



Also $\angle QPR = 90°$ [Given]

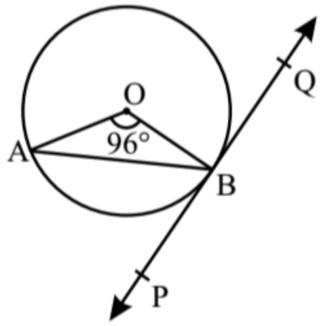
and $OQ = OR$ [Radius of the same circle]

$\therefore$ $PQOR$ is a square and so $PQ = OQ = OR = RP = 4$ cm.

Hence, the length of PQ is 4 cm.

OR

In the below figure, PQ is tangent to the circle with centre at O, at the point B. If ∠AOB = 96°, then find ∠ABP.



In ΔAOB, we have $OA = OB$ [Radii of the same circle]

$\Rightarrow \angle ABO = \angle BAO$ [∵ Angles opposite to the equal sides are equal]

Now, $\angle AOB + \angle BAO + \angle ABO = 180°$

[∵ Sum of angles of a triangle is 180°]

$\Rightarrow 96° + \angle BAO + \angle BAO = 180°$

$\Rightarrow 2\angle BAO = 84° \quad \Rightarrow \quad \angle BAO = 42°$

$\Rightarrow \angle PBO = \angle ABO = 42°$

Now, $\angle PBO = 90°$

$\Rightarrow \angle ABP + \angle ABO = 90° \quad \Rightarrow \angle ABP + 42° = 90° \quad \Rightarrow \angle ABP = 48°.$

10. Find the centre of a circle whose end points of a diameter are (4, 7) and (–8, 1).

We know that the centre of a circle is the mid-point of a diameter.

$\therefore$ The centre of the circle = $\left(\frac{4+(-8)}{2}, \frac{7+1}{2}\right) = (-2, 4)$

OR

If A (3, y) is equidistant from points P (8, –3) and Q (7, 6), find the value of y.

We have: $AP = AQ \quad \Rightarrow AP^2 = AQ^2$

$\Rightarrow (8-3)^2 + (-3-y)^2 = (7-3)^2 + (6-y)^2$

$\Rightarrow 25 + 9 + y^2 + 6y = 16 + 36 + y^2 - 12y$

$\Rightarrow 18y = 18 \quad \Rightarrow y = 1.$

11. If 4 is a root of the quadratic equation $x^2 + 2x + k = 0$, then find the value of k.

It is given that 4 is the root of the quadratic equation $x^2 + 2x + k = 0$.

$\therefore\ 4^2 + 2\,(4) + k = 0 \quad \Rightarrow \quad 16 + 8 + k = 0 \quad \Rightarrow \quad k = -24.$

OR

Show that the equation $5x^2 - 9x + 5 = 0$ is not true for any real value of x.

The given equation is $5x^2 - 9x + 5 = 0$

$\therefore\ D = (-9)^2 - 4 \times 5 \times 5 = 81 - 100 = -19 < 0$

So, the given equation has no real roots.

Hence, the given equation is not true for any real value of x.

12. Find the probability of throwing a number greater than 2 with a fair dice.

When a dice is thrown, all possible outcomes are 1, 2, 3, 4, 5, 6.

Total number of all possible outcomes = 6.

Let E be the event of getting a number greater than 2.

Then, the favourable outcomes are 3, 4, 5, 6.

Number of favourable outcomes = 4.

$\therefore \quad P(E) = \frac{4}{6} = \frac{2}{3}.$

13. A card is drawn at random from a well shuffled deck of 52 cards. Find the probability that the card drawn is neither a red card nor a queen.

Total number of cards = 52

Number of red cards = 13 + 13 = 26

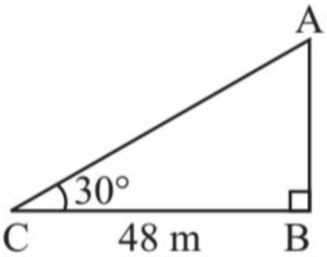
Number of queens in the remaining cards = 2

Number of cards which are neither red cards nor queens

$= \{52 - (26 + 2)\} = 24.$

$\therefore\ P$ (getting neither a red card nor a queen) = $\frac{24}{52} = \frac{6}{13}.$

14. In the given figure, the angle of elevation of the top of a tower from a point C on the ground, which is 48 m away from the foot of the tower is 30°. Find the height of the tower.



From right ΔABC, we have

$$\frac{AB}{BC} = \tan 30° \quad \Rightarrow \frac{AB}{48} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad AB = \frac{48}{\sqrt{3}} = \frac{48 \times \sqrt{3}}{48 \times \sqrt{3}} = 16\sqrt{3}$$

15. The first and the last terms of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there?

First term, $a = 17$

Common difference, $d = 9$

Last term, $l = 350$

Let there be n terms in the given AP.

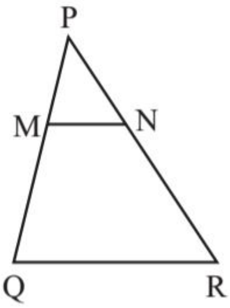
Then, $T_n = 350 \quad \Rightarrow a + (n - 1)\, d = 350$

$\Rightarrow 17 + (n - 1) \times 9 = 350 \quad \Rightarrow 17 + 9n - 9 = 350$

$\Rightarrow 9n = 342 \quad \Rightarrow \quad n = 38.$

Hence, there are 38 terms in the given AP.

16. In the given figure, MN || QR. If PN/NR = 3/7 and PM = 2.1 cm, then find MQ.



In ΔPQR, $MN \parallel QR$

$$\therefore \; \frac{PM}{MQ} = \frac{PN}{NR} \quad \text{[By Thales' theorem]}$$

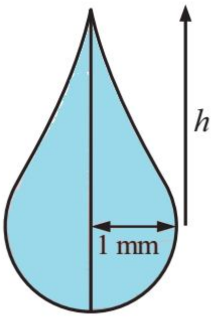
$$\Rightarrow \; \frac{2.1}{MQ} = \frac{3}{7} \quad \Rightarrow MQ = \left(\frac{2.1 \times 7}{3}\right) \text{ cm} = 49 \text{ cm.}$$

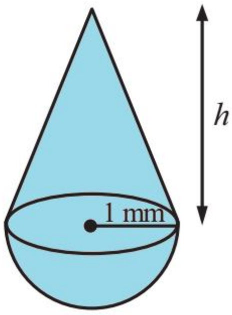
SECTION-II

Case study-based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. Case Study based-2:

In the month of December 2020, it rained heavily throughout the day over the city of Hyderabad. Anil observed the raindrops as they reached him. Each raindrop was in the shape of a hemisphere surmounted by a cone of the same radius of 1 mm. Volume of one of such drops is 3.14 mm^3. Anil collected the rain water in a pot having a capacity of 1099 cm^3. [Use $\sqrt{2} = 1.4$]





Based on the above situation, answers the following questions.
(a) Find the total height of the drop.
(i) 1 mm (ii) 2 mm (iii) 3 mm (iv) 4 mm

(b) The curved surface area of the drop is
(i) 8.74 mm^3 (ii) 9.12 mm^3 (iii) 10.68 mm^3 (iv) 12.54 mm^3

(c) As the drop fell into the pot, it changed into a sphere. What was the radius of this sphere?
(i) $(3/4)^{1/3}$ (ii) $(4/3)^{1/3}$ (iii) $3^{1/3}$ (iv) $4^{1/3}$

(d) How many drops will fill the pot completely.
(i) 260000 (ii) 280000 (iii) 320000 (iv) 350000

(e) The total surface area of a hemisphere of radius r is
(i) $2/3\ \pi r^3$ (ii) $4/3\ \pi r^3$ (iii) $2\pi r^2$ (iv) $3\pi r^2$

Ans:
(a) (ii) 2 mm **(b) (iii) 10.68 mm^3** **(c) (i) $(3/4)^{1/3}$**
(d) (iv) 350000 **(e) (iv) $3\pi r^2$**

18. Case Study based-1:
Aditya works as a librarian in Bright Children International School in Indore. He ordered for books on English, Hindi and Mathematics. He received 96 English books, 240 Hindi Books and 336 Maths books. He wishes to arrange these books in stacks such that each stack consists of the books on only one subject and the number of books in each stack is the same. He also wishes to keep the number of stacks minimum.

Based on the above situation, answers the following questions.
(a) Find the number of books in each stack.
(i) 24 (ii) 48 (iii) 54 (iv) 72

(b) Find the total number of stacks formed.

(i) 7 (ii) 10 (iii) 14 (iv) 16

(c) How many stacks of mathematics books will be formed?
(i) 7 (ii) 8 (iii) 9 (iv) 10

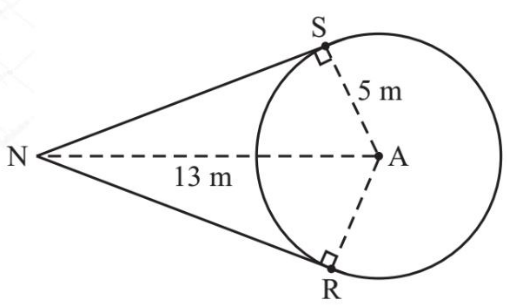
(d) If the thickness of each english book is 3 cm, then the height of each stack of english books is
(i) 120 cm (ii) 124 cm (iii) 136 cm (iv) 144 cm

(e) If each hindi book weighs 1.5 kg, then find the weight of books in a stack of hindi books.
(i) 24 kg (ii) 48 kg (iii) 72 kg (iv) 96 kg.

Ans:
(a) (ii) 48 **(b) (iii) 14** **(c) (i) 7**
(d) (iv) 144 cm **(e) (iii) 72 kg**

19. Case Study based-3:

In an international school in Hyderabad organised an Interschool Throwball Tournament for girls just after the pre-board exam. The throw ball team was very excited. The team captain Anjali directed the team to assemble in the ground for practices. Only three girls Priyanshi, Swetha and Aditi showed up. The rest did not come on the pretext of preparing for pre-board exam. Anjali drew a circle of radius 5 m on the ground. The centre A was the position of Priyanshi. She marked a point N, 13 m away from centre A as her own position. From the point N, she drew two tangential lines NS and NR and gave positions S and R to Swetha and Aditi. Anjali throws the ball to Priyanshi, Priyanshi throws it to Swetha, Swetha throws it to Anjali, Anjali throws it to Aditi, Aditi throws it to Priyanshi, Priyanshi throws it to Swetha and so on.



(a) What is the measure of ∠NSA?
(i) 30° (ii) 45° (iii) 60° (iv) 90°

(b) Find the distance between Sonia and Niharika.
(i) 8 m (ii) 12 m (iii) 15 m (iv) 18 m

(c) How far does Niharika have to throw the ball towards Riya?
(i) 18 m (ii) 15 m (iii) 12 m (iv) 8 m

(d) If ∠SNR is equal to θ, then which of the following is true?
(i) ∠ANS = 90° – θ (ii) ∠SAN = 90° – θ
(iii) ∠RAN = θ (iv) ∠RAS = 180° – θ

(e) If ∠SNR is equal to θ, then ∠NAS is equal to
(i) 90° – (θ/2) (ii) 180° – 2θ (iii) 90° – θ (iv) 90° + θ

Ans:
(a) (iv) 90° **(b) (ii) 12 m** **(c) (iii) 12 m**
(d) (iv) ∠RAS = 180° – θ **(e) (i) 90° – (θ/2)**

20. Case Study based-4:

In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that each section of each class would plant twice as many plants as the class standard. There were 3 sections of each standard from 1 to 12. So, if there are three sections in class 1 say 1A, 1B and 1C, then each section would plant 2 trees. Similarly, each section of class 2 would plant 4 trees and so on. Thus, the number of trees planted by classes 1 to 12 formed an AP given by 6, 12, 18,...

(a) What is the common difference of the AP formed?
(i) 6 (ii) 5 (iii) 3 (iv) 2

(b) What will be the nth term of the AP formed?
(i) 5n (ii) 6n (iii) 5n + 6 (iv) 6n + 6

(c) How many trees will be planted by the students of all the sections of class 8?
(i) 42 (ii) 48 (iii) 54 (iv) 60

(d) Find the total number of trees planted by class 12 students.
(i) 54 (ii) 60 (iii) 66 (iv) None of these

(e) What will be the third term from the end of the AP formed?
(i) 72 (ii) 66 (iii) 60 (iv) 54

Ans: **(a) (i) 6** **(b) (ii) 6n** **(c) (ii) 48**
(d) (iv) None of these **(e) (iii) 60**

PART – B

(Question No 21 to 26 are Very short answer Type questions of 2 mark each)

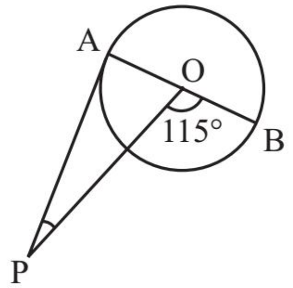
21. Prove that: $\tan\theta - \cot\theta = \dfrac{2\sin^2\theta - 1}{\sin\theta\cos\theta}$

$$\text{L.H.S.} = \tan\theta - \cot\theta = \frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta - \cos^2\theta}{\cos\theta \cdot \sin\theta}$$

$$= \frac{\sin^2\theta - (1-\sin^2\theta)}{\sin\theta \cdot \cos\theta} \quad [\because \cos^2\theta = 1 - \sin^2\theta]$$

$$= \frac{2\sin^2\theta - 1}{\sin\theta \cdot \cos\theta} = \text{R.H.S.}$$

22. In the given figure, PA is a tangent from an external point P to a circle with centre O. If ∠POB = 115°, find ∠APO.



We know that the tangent at any point to a circle is perpendicular to the radius passing through the point of contact.

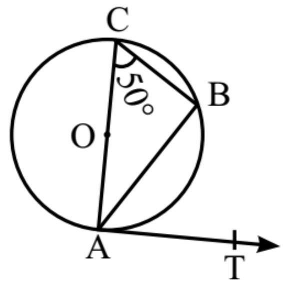
$\therefore \angle OAP = 90°$

Now, $\angle POB = \angle OAP + \angle APO$ [Exterior angle]

$\Rightarrow \angle APO = \angle POB - \angle OAP = 115° - 90° = 25°.$

OR

In the given figure, O is the centre of a circle, AOC is its diameter such that ∠ACB = 50°. If AT is the tangent to the circle at the point A then find ∠BAT.



$\angle ABC = 90°$ [∵ Angle contained in a semi-circle is right angle]

In ΔABC, $\angle ACB + \angle ABC + \angle CAB = 180°$

[∵ Sum of angles of a triangle is 180°]

$\Rightarrow 50° + 90° + \angle CAB = 180° \quad \Rightarrow \angle CAB = 40°.$

Now, $\angle CAT = \angle OAT = 90°$ [∵ The tangent at any point to a circle is perpendicular to the radius through the point of contact]

$\Rightarrow \angle CAB + \angle BAT = 90° \quad \Rightarrow 40° + \angle BAT = 90° \quad \Rightarrow \angle BAT = 50°.$

23. Show that 3 + 2√5 is irrational, given that √5 is an irrational.
Let us assume that 3 + 2√5 is a rational number such that
3 + 2√5 = a/b, where a and b are coprime numbers and b ≠ 0
⇒ 2√5 = a/b – 3 ⇒ 2√5 = (a – 3b)/b ⇒ √5 = (a – 3b)/2b
This shows (a – 3b)/2b is a rational number which contradicts the fact that √5 is an irrational number. Therefore, our assumption is wrong.
Hence 3 + 2√5 is an irrational number.

OR

Show that any number of the form 8^n, where $n \in N$ can never end with the digit 0.

If 8^n ends with 0 then it must have 5 as a factor.

But, $8^n = (2 \times 2 \times 2)^n = (2^3)^n = 2^n$ shows that 2 is the only prime factor of 8^n.

Also, we know that from the fundamental theorem of arithmetic

that the prime factorisation of each number is unique.

So, 5 is not a factor of 8^n.

Hence, 8^n can never end with the digit 0.

24. The base radius and height of a right circular solid cones are 2 cm and 8 cm respectively. It is melted and recast into spheres of diameter 2 cm each. Find the number of spheres so formed.

Radius of the cone, $r = 2$ cm

Height of the cone, $h = 8$ cm

Volume of the cone $= \frac{1}{3}\pi r^2 h = \left(\frac{1}{3} \times \pi \times 2 \times 2 \times 8\right) \text{cm}^3 = \frac{32\pi}{3}$ cm^3

Radius of each sphere, $R = 1$ cm

Volume of each sphere $= \frac{4}{3}\pi R^3 = \left(\frac{4}{3} \times \pi \times 1 \times 1 \times 1\right) \text{cm}^3 = \frac{4\pi}{3}$ cm^3

$\therefore$ Number of spheres formed $= \frac{\text{Volume of the cone}}{\text{Volume of each sphere}} = \frac{\frac{32\pi}{3}}{\frac{4\pi}{3}} = 8.$

25. ABC is a right triangle right angled at A. If AD ⊥ BC, prove that $AB^2 + CD^2 = BD^2 + AC^2$.

From right ΔADB, we have

$AB^2 = BD^2 + AD^2$ [By Pythagoras' theorem]

$\Rightarrow \quad AD^2 = AB^2 - BD^2 \quad ...(i)$

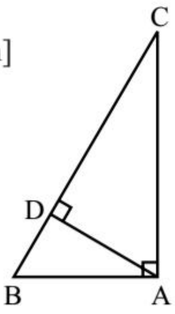
From right ΔADC, we have

$AC^2 = AD^2 + CD^2$ [By Pythagoras' theorem]

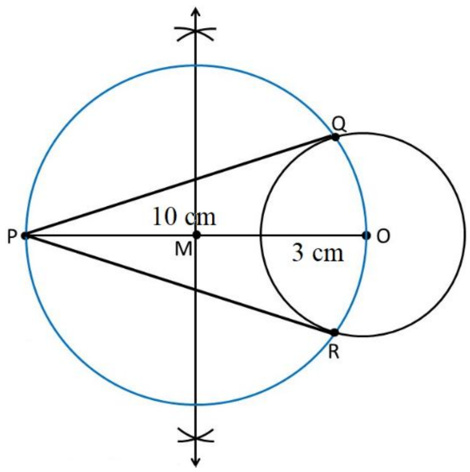
$\Rightarrow AD^2 = AC^2 - CD^2 \quad ...(ii)$

From (i) and (ii), we get $AB^2 - BD^2 = AC^2 - CD^2$

$\Rightarrow \quad AB + CD^2 = BD^2 + AC^2.$



26. Draw a circle of radius 3 cm. Take a point P at 10 cm from the centre. Construct a pair of tangents from the point P.



(Question no 27 to 33 are Short Answer Type questions of 3 marks each)

27. If cos θ + sin θ = √2 cos θ, show that cos θ – sin θ = √2 sin θ.

$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow (\cos\theta + \sin\theta)^2 = (\sqrt{2}\cos\theta)^2$ [Squaring both sides]

$\Rightarrow \cos^2\theta + \sin^2\theta + 2\cos\theta\sin\theta = 2\cos^2\theta$

$\Rightarrow \cos^2\theta - \sin^2\theta - 2\cos\theta\sin\theta = 0$

$\Rightarrow \cos^2\theta + \sin^2\theta - 2\cos\theta\sin\theta = 2\sin^2\theta$ [Adding $2\sin^2\theta$ both sides]

$\Rightarrow (\cos\theta - \sin\theta)^2 = 2\sin^2\theta \Rightarrow \cos\theta - \sin\theta = \sqrt{2\sin^2\theta} = \sqrt{2}\sin\theta$

Hence, $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

28. The sum of first n terms of an AP is $5n^2 + 3n$. If its mth term is 168, find the value of m. Also, find the 20th term of this AP.

We have $S_n = 5n^2 + 3n$ ...(*i*)

$S_{n-1} = 5(n-1)^2 + 3(n-1) = 5n^2 + 5 - 10n + 3n - 3$

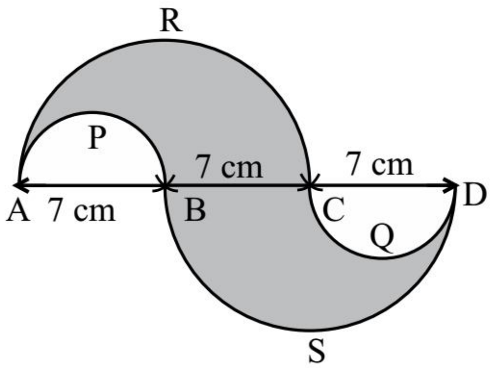
$\Rightarrow S_{n-1} = 5n^2 - 7n + 2$ ...(*ii*)

$\therefore T_n = S_n - S_{n-1} = (5n^2 + 3n) - (5n^2 - 7n + 2) = 10n - 2$

Now, $T_m = 168 \Rightarrow 10m - 2 = 168 \Rightarrow 10m = 170 \Rightarrow m = 17$.

$\therefore T_{20} = 10 \times 20 - 2 = 200 - 2 = 198$.

29. In the given figure, APB and CQD are semi-circles of diameter 7 cm each, while ARC and BSD are semi-circles of diameter 14 cm each. Find the perimeter of the shaded region.



The radius of semi circle ARC is 7cm

The radius of semi circle AP is3.5cm

The perimeter of curve is given as ARC + CQD + DSB + APB

$$\frac{22}{7} \times 7 + \frac{22}{7} \times 3.5 + \frac{22}{7} \times 3.5 + \frac{22}{7} \times 7$$

$$\frac{22}{7}(21) = 66\text{cm}$$

30. A bag contains 18 balls out of which x balls are red and remaining balls are white.

(i) If a ball is drawn at random from the bag. What is the probability that it is not red?

(ii) If 2 more white balls are put in the bag, then the probability of drawing a white ball will be 9/8 times the probability of drawing a white ball in the first case. Find the value of x.

(i) Total number of balls = 18.

Number of red balls = x.

Number of white balls = $18 - x$.

$\therefore$ P (not getting a red ball) = $\dfrac{18-x}{18}$

(ii) Total number of balls = 18 + 2 = 20

Number of white balls = $(18 - x) + 2 = 20 - x$

P (getting a white ball) = $\dfrac{20-x}{20}$

$\therefore \dfrac{20-x}{20} = \dfrac{9}{8}\left(\dfrac{18-x}{18}\right) \quad \Rightarrow \dfrac{20-x}{20} = \dfrac{18-x}{16}$

$\Rightarrow 320 - 16x = 360 - 20x \quad \Rightarrow 4x = 40 \quad \Rightarrow x = 10.$

31. Find the ratio in which the y-axis divides the line segment joining the points (5, –6) and (–1, –4). Also, find the coordinates of the point of intersection.

Any point on the y-axis is of the form $(0, y)$.

Let $P(0, y)$ divides the line segment joining the points A (5, –6) and $B(-1, -4)$ in the ratio $k : 1$.

Then, $(0, y) = \left(\dfrac{-k+5}{k+1}, \dfrac{-4k-6}{k+1}\right)$ [By section formula]

$\Rightarrow 0 = \dfrac{-k+5}{k+1} \quad \Rightarrow 0 = -k + 5 \quad \Rightarrow k = 5$

$\therefore$ The required ratio = $k : 1 = 5 : 1$

Also, $y = \dfrac{-4k-6}{k+1} = \dfrac{-4\times 5-6}{5+1} = \dfrac{-26}{6} = \dfrac{-13}{3}$

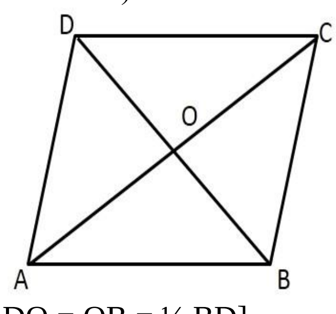
Hence, the required point of intersection is $\left(0, \dfrac{-13}{3}\right)$.

32. Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

ABCD is a rhombus with O as point of intersection of diagonals.

In ΔAOB, $\angle AOB = 90^0$ (since diagonals are perpendicular in rhombus).

By Pythagoras theorem,



$AB^2 = AO^2 + OB^2$

Similarly, $BC^2 = OC^2 + OB^2$,

$DC^2 = OD^2 + OC^2$

$DA^2 = DO^2 + OA^2$

$AB^2 + BC^2 + CD^2 + DA^2 = 2(OA^2 + OB^2 + OC^2 + OD^2)$

$= 4(AO^2 + DO^2)$

$= 4(¼\ AC^2 + ¼\ BD^2)$

[Rhombus diagonal biset each other, AO = OC = ½ AC, DO = OB = ½ BD]

$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

OR

The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, find the length of the corresponding side of the second triangle.

Let ABC and DEF be two similar triangles such that perimeter (ΔABC) = 25 cm and perimeter (ΔDEF) = 15 cm.

Let AB = 9 cm.

Since $\Delta ABC \sim \Delta DEF$ $\therefore$ $\dfrac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \dfrac{AB}{DE}$

$\Rightarrow \dfrac{25}{15} = \dfrac{9}{DE} \Rightarrow DE = \left(\dfrac{9 \times 15}{25}\right) \text{cm} = 5.4 \text{ cm}.$

Hence, the corresponding side of the other triangle is 5.4 cm.

33. If 1 is added to both the numerator and the denominator of a fraction, it becomes 4/5. If, however, 5 is subtracted from both the numerator and the denominator, the fraction becomes 1/2. Find the fraction.

Let the fraction be $\dfrac{x}{y}$. Then,

$\dfrac{x+1}{y+1} = \dfrac{4}{5} \Rightarrow 5x + 5 = 4y + 4 \Rightarrow 5x - 4y = -1$ ...(*i*)

Also, $\dfrac{x-5}{y-5} = \dfrac{1}{2} \Rightarrow 2x - 10 = y - 5 \Rightarrow 2x - y = 5$ ...(*ii*)

Multiplying (*ii*) by 4 and subtracting (*i*) from the result, we get

$8x - 5x = 20 + 1 \Rightarrow 3x = 21 \Rightarrow x = 7.$

Putting $x = 7$ in (*i*), we get

$35 - 4y = -1 \Rightarrow 4y = 36 \Rightarrow y = 9.$

Thus, $x = 7$ and $y = 9$.

Hence, the required fraction is $\dfrac{7}{9}$.

OR

Two years ago, a man was five times as old as his son. Two years later, his age will be 8 more than three times the age of his son. Find their present ages.

Let the present ages of the father and the son be x years and y years respectively. Then,

Father's age 2 years ago = $(x - 2)$ years

and son's age 2 years ago = $(y - 2)$ years

$\therefore x - 2 = 5(y - 2) \Rightarrow x - 2 = 5y - 10 \Rightarrow x - 5y = -8$ ...(*i*)

Father's age 2 years hence = $(x + 2)$ years

and son's age 2 years hence = $(y + 2)$ years.

$\therefore x + 2 = 3(y + 2) + 8 \Rightarrow x + 2 = 3y + 6 + 8 \Rightarrow x - 3y = 12$ ...(*ii*)

On subtracting (*i*) from (*ii*), we get: $2y = 20 \Rightarrow y = 10.$

On putting $y = 10$ in (*i*), we get:

$x - 5 \times 10 = -8 \Rightarrow x - 50 = -8 \Rightarrow x = 42.$

Hence, father's present age = 42 years and son's present age = 10 years.

(Question no 34 to 36 are Long Answer Type questions of 5 marks each.)

34. The median of the following data is 20.75. Find the missing frequencies x and y.

C.I.	0–5	5–10	10–15	15–20	20–25	25–30	30–35	35–40	**Total**
f	7	10	x	13	y	10	14	9	**100**

Clearly, $7 + 10 + x + 13 + y + 10 + 14 + 9 = 100 \Rightarrow x + y = 37$ ...(*i*)

Median is 20.75, which lies in 20–25. So, the median class is 20–25.

$\therefore\ l = 20, h = 5, f = y, cf = 7 + 10 + x + 13 = 30 + x,\ \dfrac{N}{2} = \dfrac{100}{2} = 50.$

Median, $M_e = l + \left\{h \times \dfrac{\frac{N}{2} - cf}{f}\right\} \Rightarrow 20.75 = 20 + 5 \times \dfrac{50 - (30 + x)}{y} \Rightarrow 0.75 = \dfrac{100 - 5x}{y}$

$\Rightarrow \dfrac{3}{4} = \dfrac{100 - 5x}{y} \Rightarrow 3y = 400 - 20x \Rightarrow 20x + 3y = 400$ ...(*ii*)

Multiplying (*i*) 3 and subtracting the result from (*ii*), we get:

$20x - 3x = 400 - 111 \Rightarrow 17x = 289 \Rightarrow x = 17.$

Putting $x = 17$ in (*i*), we get: $y = 20$

Hence, $x = 17$ and $y = 20$.

35. If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)\,x + (c^2 + d^2) = 0$ has no real roots.

The given equation is of the form $Ax^2 + Bx + C = 0$

where, $A = a^2 + b^2$, $B = 2\,(ac + bd)$ and $C = c^2 + d^2$

$$\begin{aligned} D &= B^2 - 4AC = \{2\,(ac + bd)\}^2 - 4\,(a^2 + b^2)\,(c^2 + d^2) \\ &= 4\,(a^2c^2 + b^2d^2 + 2\,abcd) - 4\,(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) \\ &= 4\,(a^2c^2 + b^2d^2 + 2\,abcd - a^2c^2 - a^2d^2 - b^2c^2 - b^2d^2) \\ &= -4\,(a^2d^2 + b^2c^2 - 2\,abcd) = -4\,(ad - bc)^2 \Rightarrow D = -4\,(ad - bc)^2 \quad ...(i) \end{aligned}$$

Now, $ad \neq bc \Rightarrow ad - bc \neq 0 \Rightarrow (ad - bc)^2 > 0 \Rightarrow -4(ad - bc)^2 < 0$ ...(*ii*)

$\therefore\ D < 0.$ [From (*i*) and (*ii*)]

Hence, the given equation has no real roots.

OR

A faster train takes 3 hours less than a slower train for a journey of 600 km. If the speed of the slower train is 10 km/hour less than that of the faster train, find the speed of the two trains.

Let the speed of the faster train be x km/hr.

Then, the speed of the slower train = $(x - 10)$ km/hr.

Time taken by the faster train to cover 600 km = $\left(\dfrac{600}{x}\right)$ hrs

Time taken by the slower train to cover 600 km = $\left(\dfrac{600}{x - 10}\right)$ hrs

$\therefore\ \dfrac{600}{x} = \dfrac{600}{x - 10} - 3 \Rightarrow \dfrac{600}{x - 10} - \dfrac{600}{x} = 3 \Rightarrow \dfrac{600x - 600\,(x - 10)}{x\,(x - 10)} = 3 \Rightarrow 6000 = 3x^2 - 30x$

$\Rightarrow 3x^2 - 30x - 6000 = 0 \qquad \Rightarrow x^2 - 10x - 2000 = 0$

$\Rightarrow x^2 - 50x + 40x - 2000 = 0 \qquad \Rightarrow x\,(x - 50) + 40\,(x - 50) = 0$

$\Rightarrow (x - 50)\,(x + 40) = 0 \Rightarrow x = 50 \quad [\because x \neq -40]$

$\therefore$ The speed of the faster train = 50 km/hr

The speed of the slower train = (50 – 10) km/hr = 40 km/hr.

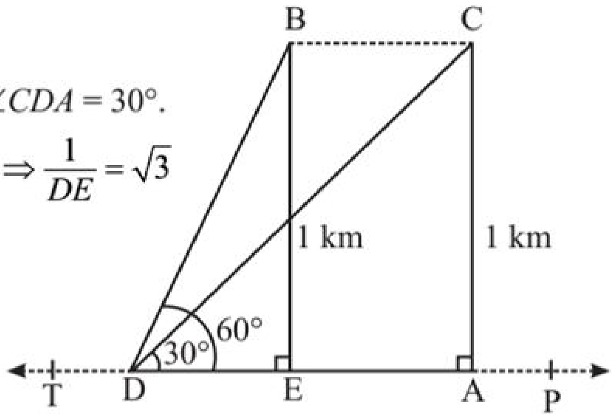
36. An aeroplane flying horizontally 1 km above the ground is observed at an elevation of 60°. After 10 seconds, its elevation is observed to be 30°. Find the speed of the aeroplane in km/hr.

Let B and C be the initial and final positions of the aeroplane.

Let PT be the horizontal line on the ground and let D be the point of observation.

Draw, $BE \perp PT$ and $CA \perp PT$.

Then, $BE = CA = 1$ km, $\angle BDE = 60°$ and $\angle CDA = 30°$.



From right ΔBED, we have $\frac{BE}{DE} = \tan 60° \Rightarrow \frac{1}{DE} = \sqrt{3}$

$$\Rightarrow DE = \frac{1}{\sqrt{3}}$$

From right ΔCAD, we have $\frac{CA}{AD} = \tan 30°$

$$\Rightarrow \frac{1}{AD} = \frac{1}{\sqrt{3}} \Rightarrow AD = \sqrt{3}$$

$$\therefore AE = AD - DE = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} = \frac{2 \times 1.732}{3}$$

Distance covered by the aeroplane in 10 seconds $= BC = AE$

$$\therefore \text{Speed of the aeroplane} = \frac{2 \times 1.732}{3}(3600) = 415.68 \text{ km/hr}$$