KENDRIYA VIDYALAYA GACHIBOWLI, GPRA CAMPUS HYD – 32
SAMPLE TEST PAPER 08 FOR CLASS X BOARD EXAM 2021
(SAMPLE ANSWERS)

Max. marks: 80 **Time Allowed: 3 hrs**

General Instruction:
1. This question paper contains two parts A and B.

Part – A:
1. It consists two sections- I and II.
2. Section I has 16 questions of 1 mark each.
3. Section II has 4 questions on case study. Each case study has 5 case-based sub-parts. An examinee is to attempt any 4 out of 5 sub-parts.

Part – B:
1. Question No 21 to 26 are Very short answer Type questions of 2 mark each,
2. Question No 27 to 33 are Short Answer Type questions of 3 marks each
3. Question No 34 to 36 are Long Answer Type questions of 5 marks each.

PART - A
SECTION-I
Questions 1 to 16 carry 1 mark each.

1. If xy = 180 and HCF(x, y) = 3, then find the LCM(x, y).

LCM × 3 = 180 ⇒ LCM = 60

OR

Determine whether $\frac{786}{1500}$ has a terminating decimal expansion or non-terminating repeating decimal expansion.

$\frac{786}{1500} = \frac{131}{250} = \frac{131}{2 \times 5^3}$

Since, the denominator is of the form $2^m \times 5^n$, so, it has a terminating decimal expansion.

2. If the sum and product of the zeroes of the polynomial $ax^2 - 6x + c$ is equal to 12 each, find the values of a and c.

Let α and β be the zeroes of the polynomial $ax^2 - 6x + c$. As per given condition,

$\alpha + \beta = -\frac{(-6)}{a} = \frac{6}{a} = 12 \Rightarrow a = \frac{1}{2}$ ...(i)

and $\alpha\beta = \frac{c}{a} = 12$ ...(ii)

From (ii), $c = 12 \times \frac{1}{2} \Rightarrow c = 6$

Thus, $a = \frac{1}{2}$ and $c = 6$.

3. If α and β are the zeroes of the polynomial we $ax^2 + bx + c$, find the value of $\alpha^2 + \beta^2$.

$\alpha + \beta = \frac{-b}{a}, \alpha\beta = \frac{c}{a}$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$\Rightarrow \alpha^2 + \beta^2 = \left(\frac{-b}{a}\right)^2 - 2\left(\frac{c}{a}\right) = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2}{a^2} - \frac{2c}{a} = \frac{b^2-2ca}{a^2}$

4. For what value of k, the pair of equations 4x – 3y = 9, 2x + ky = 11 has no solution?

We have, $4x - 3y = 9$ and $2x + ky = 11$

For no solution, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \frac{4}{2} = \frac{-3}{k} \neq \frac{9}{11} \Rightarrow 2 = \frac{-3}{k} \Rightarrow 2k = -3 \Rightarrow k = \frac{-3}{2}$.

5. Find the number of terms in the AR 7, 13, 19,....,205.

Let there be n terms in the AP, 7, 13, 19 .... 205.

Here, $a = 7$ and $d = 13 - 7 = 6$.

So, $a + (n - 1)d = 205 \Rightarrow 7 + (n - 1)\,6 = 205$

$\Rightarrow 6(n - 1) = 205 - 7 \Rightarrow 6(n - 1) = 198$

$\Rightarrow n - 1 = \frac{198}{6} = 33 \Rightarrow n = 33 + 1 = 34$.

Thus, the given AP contains 34 terms.

OR

The 4th term of an AP is zero. Prove that the 25th term of the AP is three times its 11th term.

$a + 3d = 0$ [Given] ...(i)

Now, 25th term $= a + 24d$ ...(ii)

and 11th term $= a + 10d$ ...(iii)

Now, 25th term can also written as

$= a + 24d + 2(a + 3d)$ [Since $a + 3d = 0$]

$= 3a + 30d$ [From (i)]

$= 3(a + 10d) = 3 \times$ 11th term

6. Find the values of k, if the equation $2x^2 + kx + 3 = 0$ have two equal roots.

The given equation is $2x^2 + kx + 3 = 0$

Here, $a = 2$, $b = k$ and $c = 3$

$\therefore$ D $= b^2 - 4ac = k^2 - 4 \times 2 \times 3 = k^2 - 24$

The given equation will have equal roots, if D = 0

i.e., $k^2 - 24 = 0$

$\Rightarrow k = \pm\sqrt{24} = \pm 2\sqrt{6}$

OR

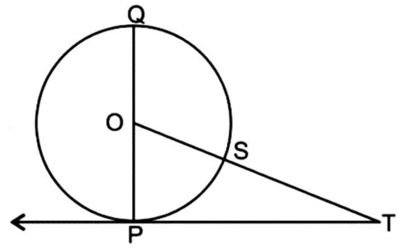
Find the value of k such that $\frac{5}{2}$ is a root of the quadratic equation $14x^2 - 27x + k = 0$.

$\frac{5}{2}$ is a root of $14x^2 - 27x + k = 0$

So, $14 \times \left(\frac{5}{2}\right)^2 - 27 \times \frac{5}{2} + k = 0 \Rightarrow 14 \times \frac{25}{4} - \frac{135}{2} + k = 0$

$\Rightarrow \frac{175}{2} - \frac{135}{2} + k = 0 \Rightarrow k = \frac{-40}{2} \Rightarrow k - 20$

7. In the below figure, TP is a tangent to the circle with centre O. If $\angle TOQ = 120°$, find the diameter of the circle, when OT = 10 cm.



$\angle TOQ = 120°$

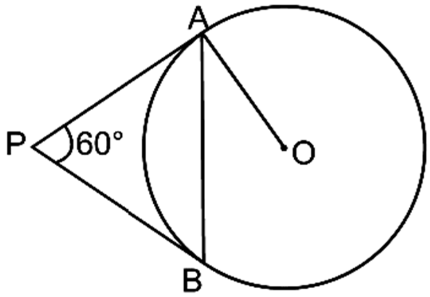
Therefore, $\angle POT = 180° - 120° = 60°$ [Linear pair]

From right ΔPOT, $\frac{OP}{OT} = \cos 60°$

$\Rightarrow \frac{OP}{10} = \frac{1}{2} \Rightarrow OP = \frac{10}{2} = 5$ cm

So, diameter of the circle = 2 × 5 cm = 10 cm.

8. In the given figure, PA and PB are tangents to the circle with centre O. If $\angle APB = 60°$, then find $\angle OAB$.



$\angle 1 = \angle 2$

Now, $\angle 1 + \angle 2 + \angle APB = 180°$

$\Rightarrow \angle 1 + \angle 1 + 60° = 180°$

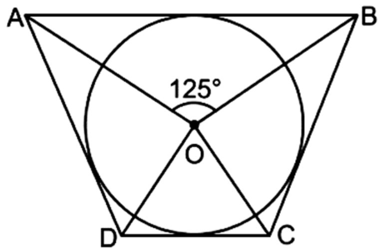
$\Rightarrow 2\angle 1 = 180° - 60° = 120° \Rightarrow \angle 1 = \frac{120°}{2} = 60°$

$\angle 1 + \angle OAB = 90° \Rightarrow 60° + \angle OAB = 90°$

$\Rightarrow \angle OAB = 90° - 60° = 30°$.

OR

In the given figure, if $\angle AOB = 125°$, then find $\angle COD$.



We know that a quadrilateral circumscribing a circle subtends supplementary angles at the centre of the circle

$\therefore \angle AOB + \angle COD = 180°$

$\Rightarrow 125° + \angle COD = 180° \Rightarrow \angle COD = 180° - 125° = 55°$

9. Let $\Delta ABC \sim \Delta DEF$ and their areas be 64 cm^2 and 121 cm^2 respectively. If EF = 15.4 cm, find BC.

We have, $\frac{Area(\Delta ABC)}{Area\ (\Delta DEF)} = \frac{BC^2}{EF^2}$

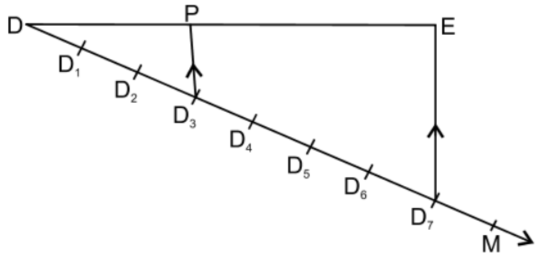
So, $\frac{64}{121} = \frac{BC^2}{(15.4)^2}$

$\Rightarrow \frac{8}{11} = \frac{BC}{15.4} \Rightarrow BC = \left(\frac{8}{11} \times 15.4\right) \Rightarrow BC = 11.2$ cm

10. In the figure, D_1, D_2, D_3 .... are points on ray DM at equal distances and $D_3P \parallel D_7E$. What will be the ratio of DE to DP?

P divides DE in the ratio 3 : 4.

Ratio of DE to DP = 7 : 3.



11. Find the value of sin 60° cos 30° – cos 60° sin 30°

$\sin 60° \cos 30° - \cos 60° \sin 30°$

$$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}.$$

12. If $\tan\alpha = \sqrt{3}$ and $\tan\beta = \frac{1}{\sqrt{3}}$, $0 < \alpha, \beta < 90°$, find the value of $\cot(\alpha + \beta)$.

$\tan \alpha = \sqrt{3} = \tan 60° \Rightarrow \alpha = 60°$

$\tan \beta = \frac{1}{\sqrt{3}} = \tan 30° \Rightarrow \beta = 30°$

$\therefore \cot(\alpha + \beta) = \cot(60° + 30°) = \cot 90° = 0.$

13. If the area of a circle is equal to sum of the areas of two circles of diameters 10 cm and 24 cm, calculate the diameter of the larger circle (in cm).

$\pi R^2 = \pi r_1^2 + \pi r_2^2$

$\Rightarrow \pi R^2 = \pi(r_1^2 + r_2^2)$ $\qquad \left[r_1 = \frac{10}{2} = 5 \text{ cm}, r_2 = \frac{24}{2} = 12 \text{ cm} \right]$

$\Rightarrow R^2 = 5^2 + 12^2 = 25 + 144$

$\Rightarrow R^2 = 169 \Rightarrow R = 13$ cm

$\therefore$ Diameter = 2(13) cm = 26 cm.

14. Find the number of solid spheres, each of diameter 6 cm that can be made by melting a solid metal cylinder of height 45 cm and diameter 4 cm.

$$\text{Number of solid spheres} = \frac{\text{Volume of cylinder}}{\text{Volume of one solid sphere}} = \left(\frac{\pi(2)^2(45)}{\frac{4}{3}\pi(3)^3} \right)$$

$$= \frac{2 \times 2 \times 45}{\frac{4}{3} \times 3 \times 3 \times 3} = 5 \qquad \left[\text{Volume of cylinder} = \pi r^2 h \text{ and volume of sphere} = \frac{4}{3}\pi r^3 \right]$$

15. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere?

Let radius of hemisphere be r units

Volume of hemisphere = S.A. of hemisphere

$\Rightarrow (2/3)\pi r^3 = 3\pi r^2 \Rightarrow r = 9/2$

or diameter = 9 units

16. Find the probability of getting doublet when two dice are thrown simultaneously?

Ans: 1/6

OR

The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap?

$$\text{P (a rotten apple)} = \frac{\text{No. of rotten apples}}{\text{Total apples}} \Rightarrow 0.18 = \frac{\text{No. of rotten apples}}{900}$$

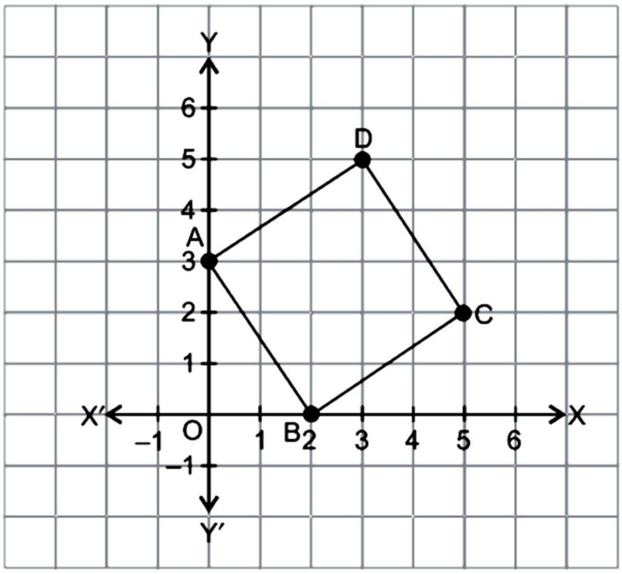
$\therefore$ No. of rotten apples = 900 × 0.18 = 162.

SECTION-II

Case study-based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. Case Study based-1:

Four friends Ram, Shyam, Lakshman and Mohan are playing with ball in a park on Sunday. Another student sitting in the park and completing his Maths Art integrated Project. He plots their positions A, B, C and D on the graph. the figure formed is the in the shape of a parallelogram with one of its vertices intersecting x-axis at 2 and another vertex intersecting the y-axis at 3.



Please give answers to these questions.

(a) Write the coordinates of A and B.

(i) A(3, 3), B(2, 2) (ii) A(0, 3), B(0, 2)

(iii) A(0, 3), B(2, 0) (iv) A(3, 0), B(0, 2)

Ans: (iii) A(0, 3), B(2, 0)

(b) Write the coordinates of C and D.

(i) C(5, 2), D(3, 5) (ii) C(5, 2), D(3, 5)

(iii) C(2, 5), D(5, 3) (iv) C(5, 2), D(3, 4)

Ans: (ii) C(5, 2), D(3, 5)

(c) Distance of point D from the origin is:

(i) $\sqrt{17}$ (ii) $\sqrt{34}$ (iii) 5 (iv) none of these

Ans: (ii) $\sqrt{34}$

(d) The distance from A to B is:

(i) $\sqrt{13}$ (ii) $\sqrt{15}$ (iii) $\sqrt{7}$ (iv) $\sqrt{5}$

Ans: (i) $\sqrt{13}$

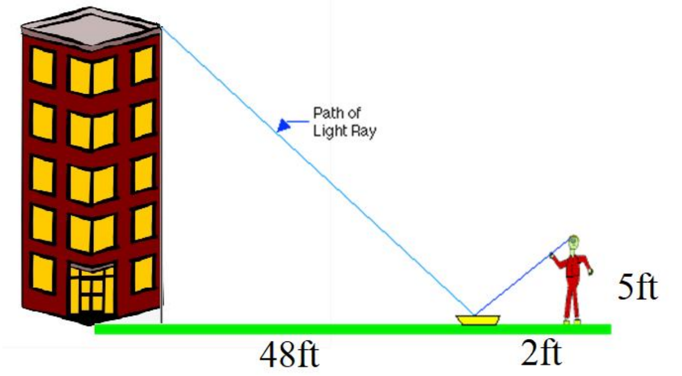
(e) In parallelogram ABCD, if length of side AB is 6 cm, then the length of side CD is :

(i) 6 cm (ii) 12 cm (iii) 3 cm (iv) 4 cm

Ans: (i) 6 cm

18. Case Study based-2:

One day Rahul decided to measure the height of the building in his locality. He is 5 feet tall. He places a mirror on the ground and moves until he can see the top of a building. At the instant when Rahul is 2 feet from the mirror, the building is 48 feet from the mirror



(a) The height of the building is :

(i) 120 feet (ii) 96 feet (iii) 40 feet (iv) 180 feet

Ans: (i) 120 feet

(b) If the distance between mirror and Rahul is decreased, what happens to the image of the building?

(i) No change (ii) Increased (iii) Decreased (iv) Same

Ans: (iii) Decreased

(c) If ΔABC ~ ΔDEF, then which of the following is true?

(i) $\frac{AB}{BC} = \frac{DE}{DF}$ (ii) ∠ABC = ∠FED

(iii) AC x DE = BC x DF (iv) None of these

Ans: (ii) ∠ABC = ∠FED

(d) Which of the following is not a similarity criterion?

(i) AA (ii) SAS (iii) AAA (iv) RHS

Ans: (iv) RHS

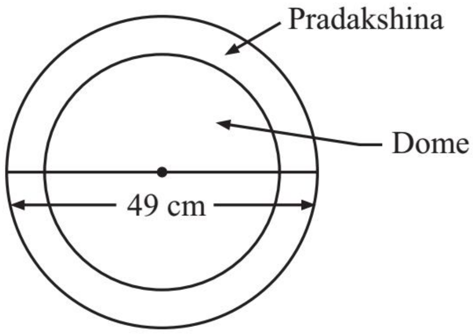
(e) Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio:

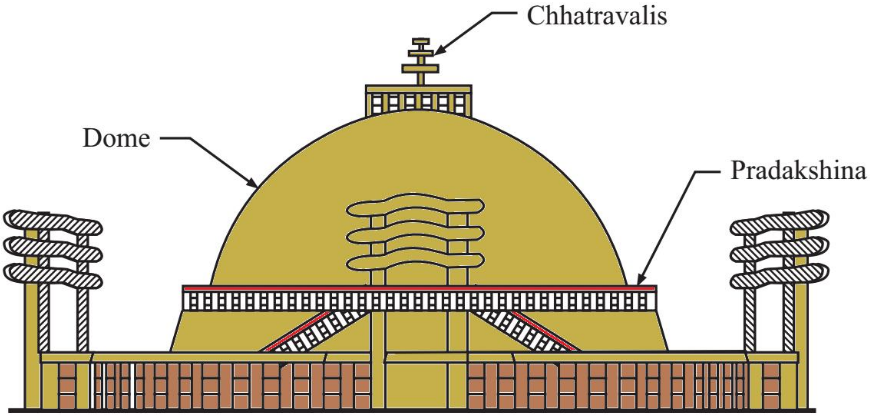
(i) 2 : 3 (ii) 4 : 9 (iii) 81 : 16 (iv) 16 : 81

Ans: (iv) 16 : 81

19. Case Study based-3:

Sanchi Stupa is a Buddhist complex in Raisen District of the state of Madhya Pradesh. A renowned architect prepared a small scale model which is an exact replica of the Stupa. The diameter of the hemispherical dome is 42 cm. The dome is filled with clay. Three chhatravalis are fixed on the top of the dome. These chhatravalis have radii 2.1 cm, 1.4 cm and 0.7 cm respectively. One Oopri Pradakshina Path is attached all around the dome. The outer diameter of this path is 49 cm.





(a) The volume of clay used to prepare the dome is

(i) 16816 cm^3 (ii) 17151 cm^3 (iii) 19404 cm^3 (iv) 21105 cm^3

Ans: (iii) 19404 cm^3

(b) The architect was asked to cover all the chhatravalis by a conical chhatra (umbrella) having radius same as that of the chhatravali and height 2.4 cm. Find the area of the silk cloth required to prepare the chhatra.

(i) 4.2 cm^2 (ii) 5.5 cm^2 (iii) 6.25 cm^2 (iv) 7.5 cm^2

Ans: (ii) 5.5 cm^2

(c) The chhatravalis are to be gold plated. Taking the thickness of each chhatravali to be negligible, find the area of the largest chhatravali to be gold plated.

(i) 18.56 cm^2 (ii) 21.14 cm^2 (iii) 27.72 cm^2 (iv) 33.36 cm^2

Ans: (iii) 27.72 cm^2

(d) Find the cost of fixing crystals on the surface of the dome at a rate of Rs. 5 per cm^2.

(i) Rs. 13860 (ii) Rs. 14445 (iii) Rs. 17475 (iv) Rs. 18360

Ans: (i) Rs. 13860

(e) Find the area of the Oopri Pradakshina Path.

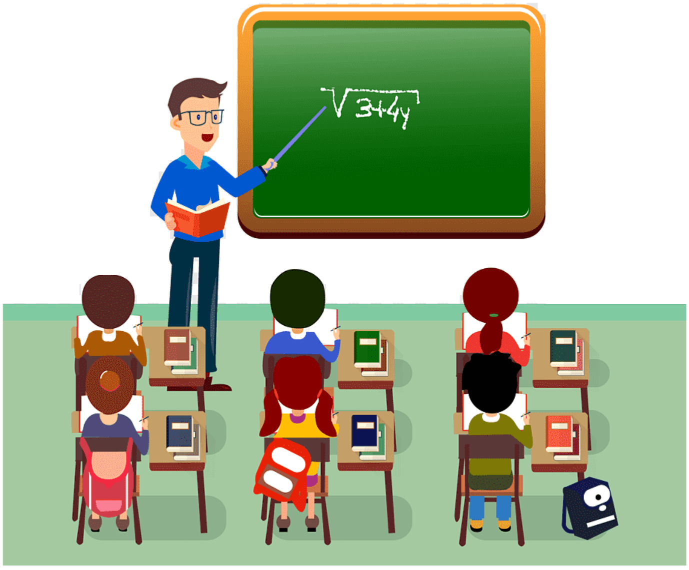
(i) 201.1 cm^2 (ii) 363.5 cm^2 (iii) 500.5 cm^2 (iv) 801.1 cm^2

Ans: (iii) 500.5 cm^2

20. Case Study based-4:

Mr. Kumar is a Maths teacher who is working in KV Gachibowli Hyderabad. In his class X, total 80 students are there. He decided to teach them as per their capabilities. So, he conducted one revision test on the basis of class IX result. The maximum marks were 50. There were 12 students who scored less than 10 marks. Shruthi who got 3 marks was handed over a red card as an intimation to work hard for one month and show improvement, as she scored the least in the class. Anish was presented a badge of honour for scoring the highest in the class. He scored 48 marks. Best performer badge given to Anish. Mr. Kumar prepared a frequency distribution table for the data of the marks obtained by the students in the revision test as follows:

Marks	Number of students
0 – 10	12
10 – 20	16
20 – 30	21
30 – 40	13
40 – 50	18



(a) Find the lower limit of modal class of the frequency distribution obtained by Sudhakar.
(i) 10 (ii) 20 (iii) 30 (iv) 40
Ans: (ii) 20

(b) Find the median class of the distribution.
(i) 10–20 (ii) 20–30 (iii) 30–40 (iv) 40–50
Ans: (ii) 20–30

(c) Find the mean marks obtained by the students.
(i) 23.25 (ii) 24.25 (iii) 26.125 (iv) 31.375
Ans: (iii) 26.125

(d) Find the range of the marks obtained by the students.
(i) 31 (ii) 37.25 (iii) 41.25 (iv) 45
Ans: (iv) 45

(e) Mr. Kumar formed Section A for those who scored above 40; Section B for those who scored between 30 and 40; Section C for those who scored between 20 and 30 and Section D for those who scored below 20. How many students were there in Section D.
(i) 12 (ii) 16 (iii) 28 (iv) 49
Ans: (iii) 28

PART – B

(Question No 21 to 26 are Very short answer Type questions of 2 mark each)

21. Check whether 4^n can end with the digit 0 for any natural number n.

$4^n = (2^2)^n = 2^{2n}$

The only prime in the factorization of 4^n is 2.

There is no other prime in the factorization of $4^n = 2^{2n}$

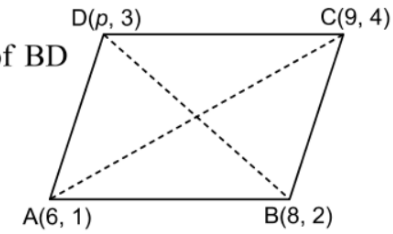
[By uniqueness of the Fundamental Theorem of Arithmetic].

5 does not occur in the prime factorization of 4^n for any n.

Therefore, 4^n does not end with the digit zero for any natural number n.

22. If the points A(6, 1), B(8, 2), C(9, 4) and D(p, 3) are the vertices of a parallelogram, taken in order, find the value of p.

Since, diagonals of a parallelogram bisect each other, so coordinates of mid-point of AC = coordinates of mid-point of BD



$\Rightarrow \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+p}{2}, \frac{2+3}{2}\right)$

$\Rightarrow \left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+p}{2}, \frac{5}{2}\right) \Rightarrow 8 + p = 15 \Rightarrow p = 7$

OR

Find the coordinates of the point which divides the join of (-1, 7) and (4, -3) in the ratio 2 : 3.

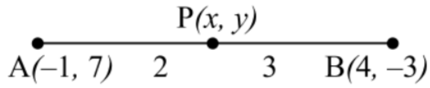
Let P(x, y) be the required point.

Then, $x = \frac{2 \times 4 + 3 \times (-1)}{2+3}$ and $y = \frac{2 \times (-3) + 3 \times 7}{2+3}$

So, $x = \frac{8-3}{5}$ and $y = \frac{-6+21}{5}$

or $x = 1$ and $y = 3$.

So, the coordinates of P are (1, 3)



23. Solve the following quadratic equation for x : $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right) x + 1 = 0$

$x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$

$\Rightarrow x^2 + \frac{ax}{a+b} + \frac{a+b}{a}x + 1 = 0 \Rightarrow x\left(x + \frac{a}{a+b}\right) + \frac{a+b}{a}\left(x + \frac{a}{a+b}\right) = 0$

$\Rightarrow \left(x + \frac{a+b}{a}\right)\left(x + \frac{a}{a+b}\right) = 0 \Rightarrow x = -\frac{a+b}{a}$ or $x = -\frac{a}{a+b}$.

24. Draw a circle of radius 4 cm with centre 0. Draw a diameter POQ. Through P, construct a tangent to the circle.

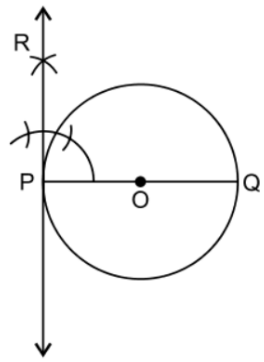
Steps of construction :

(i) Draw a circle of radius 4 cm with centre O.

(ii) Draw a diameter POQ.

(iii) At P, draw a perpendicular PR to PQ.

Then, PR is the required tangent to the circle.



25. Prove the identity: $\frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta} = 1 - \sin\theta\cos\theta$

L.H.S. $= \frac{\sin^3\theta + \cos^3\theta}{\sin\theta + \cos\theta}$

$= \frac{(\sin\theta + \cos\theta)(\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta)}{(\sin\theta + \cos\theta)}$ $[\because a^3 + b^3 = (a+b)(a^2 + b^2 - ab)]$

$= 1 - \sin\theta\cos\theta =$ R.H.S. $[\because \sin^2\theta + \cos^2\theta = 1]$

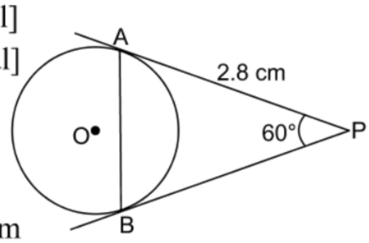
OR

Evaluate: $\frac{5\cos^2 60^0 + 4\sec^2 30^0 - \tan^2 45^0}{\sin^2 30^0 + \cos^2 30^0}$

$$\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \frac{5\times\frac{1}{4} + 4\times\frac{4}{3} - 1}{\frac{1}{4}+\frac{3}{4}} = \frac{\frac{1}{12}(15+64-12)}{\frac{1+3}{4}} = \frac{\frac{1}{12}\times 67}{\frac{4}{4}} = \frac{67}{12}.$$

26. If PA and PB are two tangents drawn to a circle from an external point P such that PA = 2.8 cm and ∠APB = 60°. Find the length of chord AB.



PA = PB [Tangents from an external point are equal]

So, ∠PAB = ∠PBA [Angles opposite to equal sides are equal]

Now, ∠PAB + ∠PBA + ∠APB = 180°

⇒ ∠PAB + ∠PBA + 60° = 180°

⇒ 2∠PAB = 120° ⇒ ∠PAB = 60°. Thus, ∠PBA = 60°

Hence, PAB is an equilateral triangle. So, chord AB = PA = 2.8 cm

(Question no 27 to 33 are Short Answer Type questions of 3 marks each)

27. Prove that 2 + 3√5 is an irrational number.

Let us assume, to the contrary, that $2 + 3\sqrt{5}$ is rational.

So, $2 + 3\sqrt{5} = \frac{a}{b}$, where

a and b are co-prime and $b \neq 0$,

Rearranging the above equation, we get

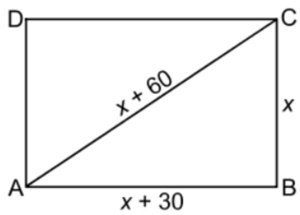
$\Rightarrow 3\sqrt{5} = \frac{a}{b} - 2 \Rightarrow 3\sqrt{5} = \frac{a-2b}{b} \Rightarrow \sqrt{5} = \frac{a-2b}{3b}$

Since a and b are integers, so $\frac{a-2b}{3b}$ is rational and so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. Thus our assumption was wrong.

So, we conclude that $2 + 3\sqrt{5}$ is irrational.

28. The diagonal of a rectangular field is 60 m more than the shorter side. If the longer side is 30 m more than the shorter side, find the sides of the field.



Let the shorter side of the rectangular field ABCD, be BC = x m.

Then, AC = $(x + 60)$ m and AB = $(x + 30)$ m

By Pythagoras theorem, we have :

$AC^2 = BC^2 + AB^2$

So, $(x + 60)^2 = x^2 + (x + 30)^2$

$\Rightarrow x^2 + 120x + 3600 = x^2 + x^2 + 60x + 900$

$\Rightarrow x^2 - 60x - 2700 = 0$

So, $x^2 - 90x + 30x - 2700 = 0$

$\Rightarrow x(x - 90) + 30(x - 90) = 0 \Rightarrow (x - 90)(x + 30) = 0$

$\Rightarrow x = 90$ or $x = -30$

$\therefore x = 90$ [∵ Side of a rectangle can never be negative]

Hence, the sides of the field are 120 m and 90 m.

OR

If the equation $(1 + m^2)n^2 x^2 + 2mn\,cx + (c^2 - a^2) = 0$ in x has equal roots, prove that $c^2 = a^2 (1 + m^2)$.

$(1 + m^2)\, n^2 x^2 + 2mn\, cx + (c^2 - a^2) = 0$, it has equal roots.

$\therefore$ D = 0 or $b^2 = 4ac$

$\Rightarrow (2mnc)^2 = 4[(1 + m^2)n^2]\,(c^2 - a^2)$

$\Rightarrow 4m^2 n^2c^2 = 4n^2 (1 + m^2)(c^2 - a^2)$

$\Rightarrow m^2c^2 = c^2 - a^2 + m^2c^2 - m^2a^2$

$\Rightarrow 0 = c^2 - a^2 - m^2a^2 \Rightarrow c^2 = a^2 (1 + m^2)$.

29. Find the area of the segment of a circle whose radius is 10 cm and the angle subtended by the corresponding chord at the centre is 30°.

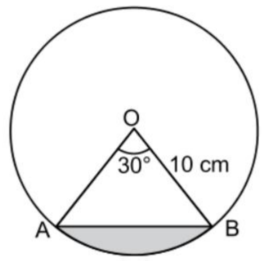
Area of segment = Area of sector – Area of triangle

$= \frac{30°}{360°}(\pi \times 10^2) - \frac{1}{2} \times 10 \times 10 \times \sin 30°$

$= \left(\frac{1}{12} \times 3.14 \times 100 - 50 \times \frac{1}{2}\right)$ cm^2

$= \left(\frac{314}{12} - 25\right)\text{cm}^2 = \left(\frac{157}{6} - 25\right)\text{cm}^2$

$= \left(\frac{157 - 150}{6}\right)\text{cm}^2 = \frac{7}{6}\text{cm}^2$



30. In ΔABC, if AP $\perp$ BC and $AC^2 = BC^2 - AB^2$, then prove that $PA^2 = PB \times CP$.

In the figure,

$AC^2 = BC^2 - AB^2$ [Given]

So, $AC^2 + AB^2 = BC^2$

Hence, $\angle BAC = 90°$ [By converse of Pythagoras theorem]

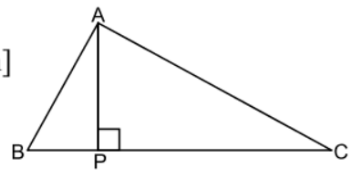
Now, in ΔABP and ΔCAP, we get

$\angle BAP = \angle 90° - \angle B = \angle ACP$

and $\angle APB = \angle CPA$ [Each = 90°]

So, $\Delta ABP \sim \Delta CAP$ [AA similarity]

$\Rightarrow \frac{PA}{PC} = \frac{PB}{PA} \Rightarrow PA^2 = PB \times PC$



OR

In an equilateral triangle ABC, D is a point on side BC such that BD = 1/3 BC. Prove that $9AD^2 = 7AB^2$.

Draw AM $\perp$ BC

$BD = \frac{1}{3}BC,\ BM = \frac{1}{2}BC$

In ΔABM,

$AB^2 = AM^2 + BM^2$

$= AM^2 + (BD + DM)^2$

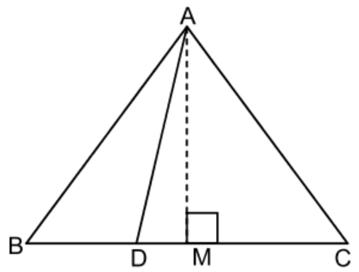
$= AM^2 + DM^2 + BD^2 + 2BD.DM$ [In ΔADM, $AM^2 + DM^2 = AD^2$]

$= AD^2 + BD^2 + 2BD(BM - BD)$

$= AD^2 + \left(\frac{BC}{3}\right)^2 + 2 \cdot \frac{BC}{3} \cdot \left(\frac{BC}{2} - \frac{BC}{3}\right)$

$= AD^2 + 2\frac{AB^2}{9} \Rightarrow 9AB^2 - 2AB^2 = 9AD^2$

Hence, $7AB^2 = 9AD^2$.



31. Two different dice are thrown together. Find the probability that the numbers obtained have
(i) even sum, and
(ii) even product.

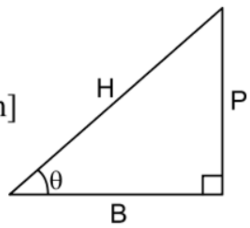
When two different dice are thrown together
Total outcomes = 6 × 6 = 36
(i) For even sum—Favourable outcomes are
(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5),
(4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)
No. of favourable outcomes = 18
P (even sum) = Favourable outcomes/Total outcomes = 18/36 = 1/2
(ii) For even product—Favourable outcomes are
(1, 2), (1, 4), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 2), (3, 4), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6),
(5, 2), (5, 4), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6).
No. of favourable outcomes = 27
P(even product) = Favourable outcomes/Total outcomes = 27/36 = 3/4

32. If $\sin\theta = \frac{12}{13}$, $0^0 < \theta < 90^0$, find the value of $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta\cos\theta} \times \frac{1}{\tan^2\theta}$.

$\sin\theta = \frac{12}{13} \Rightarrow \frac{P}{H} = \frac{12}{13}$

Let P = 12K, H = 13K

$P^2 + B^2 = H^2$ [Pythagoras theorem]



$\Rightarrow (12K)^2 + B^2 = (13K)^2$

$\Rightarrow 144K^2 + B^2 = 169K^2$

$\Rightarrow B^2 = 169K^2 - 144K^2 = 25K^2 \Rightarrow B = 5K$

$\therefore \cos\theta = \frac{B}{H} = \frac{5K}{13K} = \frac{5}{13}$

$\tan\theta = \frac{P}{B} = \frac{12K}{5K} = \frac{12}{5}$

Now, $$\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta \,.\, \cos\theta} \times \frac{1}{\tan^2\theta} = \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)} \times \frac{1}{\left(\frac{12}{5}\right)^2} = \frac{\frac{144-25}{169}}{\frac{120}{169}} \times \frac{25}{144}$$

$$= \frac{119}{120} \times \frac{25}{144} = \frac{595}{3456}.$$

33. The mean of the following frequency distribution is 62.8 and the sum of frequencies is 50. Find the missing frequencies f_1 and f_2:

Class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120
Frequency	5	f_1	10	f_2	7	8

Class (C.I.)	Frequency (f_i)	x_i	$f_i x_i$
0-20	5	10	50
20-40	f_1	30	$30f_1$
40-60	10	50	500
60-80	f_2	70	$70f_1$
80-100	7	90	630
100-120	8	110	880
	$\Sigma f_i = 50$		$\Sigma f_i x_i = 2060 + 30f_1 + 70f_2$

$5 + f_1 + 10 + f_2 + 7 + 8 = 50$ [Given]

$\Rightarrow 30 + f_1 + f_2 = 50 \Rightarrow f_2 = 20 - f_1$ ...(i)

$\therefore$ Mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i}$

$\therefore \dfrac{62.8}{1} = \dfrac{2060 + 30f_1 + 70f_2}{50}$ $\because$ Mean = 62.8 [Given] $\Rightarrow 2060 + 30f_1 + 70f_2 = 3140$

$\Rightarrow 30f_1 + 70f_2 = 3140 - 2060 = 1080 \Rightarrow 3f_1 + 7f_2 = 108$

$\Rightarrow 3f_1 + 140 - 7f_1 = 108$ [From (i)]

$\Rightarrow -4f_1 = 108 - 140 = -32$. $\Rightarrow f_1 = 8$

Putting the value of f_1 into (i), we get $f_2 = 20 - 8 = 12 \Rightarrow f_2 = 12$

Hence $f_1 = 8$ and $f_2 = 12$.

(Question no 34 to 36 are Long Answer Type questions of 5 marks each.)

34. On reversing the digits of a two digit number, the number obtained is 9 less than three times the original number. If difference of these two numbers is 45, find the original number

Let unit's digit be x and ten's digit be y.

$\therefore$ Original number $= x + 10y$. Reversed number $= 10x + y$

According to the question,

$10x + y = 3(x + 10y) - 9$

$\Rightarrow 10x + y = 3x + 30y - 9 \Rightarrow 10x + y - 3x - 30y = -9 \Rightarrow 7x - 29y = -9$ …(i)

Also, $10x + y - (x + 10y) = 45$

$\Rightarrow 9x - 9y = 45 \Rightarrow x - y = 5$ …[Dividing both sides by 9]

$\Rightarrow x = 5 + y$ …(ii)

From (i) and (ii) we have,

$7(5 + y) - 29y = -9$ …[From (ii)

$\Rightarrow 35 + 7y - 29y = -9$

$-22y = -9 - 35$

$-22y = -44 \Rightarrow y = \dfrac{44}{22} = 2$

Putting the value of y in (ii), we have

$x = 5 + 2 = 7$

$\therefore$ Original number $= x + 10y = 7 + 10(2) = 27$.

35. If the ratio of the sum of the first n terms of two A.P.s is (7n + 1) : (4n + 27), then find the ratio of their 9th terms.

Ratio of sum of first n terms of two A.P.s are

$$\frac{\frac{n}{2}\left[2a+(n-1)d\right]}{\frac{n}{2}\left[2A+(n-1)D\right]} = \frac{7n+1}{4n+27}$$

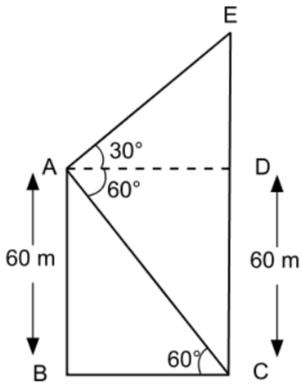
Put $n = 17 \Rightarrow \quad \frac{2a+(16)d}{2A+(16)D} = \frac{120}{95}$

$$\frac{2a+(16)d}{2A+(16)D} = \frac{120}{95} = \frac{24}{19}$$

$$\frac{a+8d}{A+8D} = \frac{24}{19}$$

Hence ratio of 9^{th} terms of two A.P.s is 24 : 19

36. The angles of elevation and depression of the top and bottom of a light house from the top of a 60 m high building are 30° and 60° respectively. Find: (i) the difference between the heights of the light house and the building. (ii) the distance between the bases of light house and the building.



Let AB = 60 m be the building

and CE be the lighthouse.

In right ΔABC,

$\tan 60° = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{60}{BC} \Rightarrow \sqrt{3}\,BC = 60$

$\Rightarrow BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$ ..(i)

$= 20(1.732) = 34.64$ m

AD = BC [Opp. sides of a rectangle]

$\therefore$ AD = $20\sqrt{3}$ ...(ii) [From (i)]

In right. ΔADE, we have

$\tan 30° = \frac{DE}{AD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{DE}{20\sqrt{3}}$ ...[From (ii)]

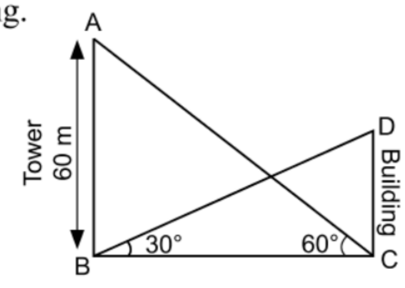
$\Rightarrow \sqrt{3}\,DE = 20\sqrt{3} \quad \Rightarrow DE = 20$ m

$\therefore$ (i) Difference between their heights = 20 m

and (ii) Distance between their bases = 34.64 m

OR

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 60 m high, find the height of the building.



Let AB = 60 m be the tower and let CD be the building.

In right ΔABC, $\tan 60° = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{60}{BC} \quad \Rightarrow \sqrt{3}\,BC = 60$

$\Rightarrow BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3}$ m ...(*i*)

In right ΔBCD, $\tan 30° = \frac{CB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{CD}{20\sqrt{3}}$

$\Rightarrow CD = 20$

∴ Height of the building, CD = 20 m